Algebra II: Homework assignment 8
Due date: April 21

1. Let f be the polynomial $x^3 - 5$.
(i) Show that the polynomial f is irreducible over $\mathbb{Q}$.
(ii) Show that the Galois group of the splitting field K of f is the symmetric group S_3.
(iii) Find the fixed field in K of the alternating subgroup $A_3 \subset S_3 = Gal(K/\mathbb{Q})$.

2. Let f be an irreducible polynomial of degree 6 over a field F, and let K be a quadratic extension of F. Prove or disprove: Either f is irreducible over K, or else f is a product of two irreducible cubic polynomials.

3. (a) Let $f \in F[x]$ be an irreducible quartic polynomial over a field F, and let K be the splitting field of f. What are the possible Galois groups of K/F?
(b) Find the Galois group of the polynomial $x^4 + 8x + 12$ over $\mathbb{Q}$.

4. Let a be an element of a field F, and let p be a prime. Suppose that the polynomial $x^p - a$ is reducible in $F[x]$. Prove that it has a root in F.

5. Let K be a field with p^n elements. Prove that the Frobenius map defined by $\varphi(x) = x^p$ is a linear transformation of K, when K is viewed as a vector space over the prime field $\mathbb{F}_p$, and determine its eigenvectors, eigenvalues and characteristic polynomial.