Algebra II: Homework assignment 2
Due date: February 16

1. Find the minimal and characteristic polynomials of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & -1 \\ 0 & 2 & 1 & -1 \\ 1 & -1 & 2 & -1 \\ 1 & -1 & 0 & 1 \end{pmatrix}$$

Find its Jordan and rational canonical forms.

2. Determine the Jordan canonical form for the $n \times n$ matrix over $\mathbb{F}_p$ whose entries are all equal to 1 except that the entries along the main diagonal are all equal to 0.

Lagrange interpolation formula:

3. (a) Let V be the space of polynomials of degree less than 3 with real coefficients. Let f_0, f_1, f_2 and g be the linear functionals on V defined by the formulas:

$$f_i(p) = p(i), \quad g(p) = p'(0),$$

fo any $p \in V$ and $i = 0, 1, 2$. Show that f_0, f_1 and f_2 form a basis in V and find the decomposition of g in this basis.

(b) Find a polynomial f of degree 5 such that $f(0) = f^{(iv)}(0) = 1$, $f'(0) = f'''(0) = 2$ and $f''(0) = 0$.

Tensor, exterior and symmetric algebras.

4. Let V be a vector space. Construct a canonical isomorphism between $V \otimes V^*$ and $\mathrm{Hom}(V, V)$.

5. Let V be a vector space over a field $\mathbb{F}$. Show that if $\mathrm{char}(\mathbb{F}) \neq 2$ then every bilinear form on V can be written uniquely as a sum of a symmetric and an antisymmetric bilinear forms. Does this statement remain true for n–linear forms if $n > 2$?

6. Let V be a vector space, and $A : V \to V$ a linear operator. Prove that the coefficient with x^{n-i} of the characteristic polynomial $\det(xI - A)$ is equal to $(-1)^i \mathrm{Tr}(\Lambda^i A)$.