

Algebra II: Optional practice problems on Representation Theory

Let G be a finite group. For any two class functions χ, χ' on G , define their inner product by the formula

$$(\chi, \chi') = \frac{1}{|G|} \sum_{g \in G} \overline{\chi(g)} \chi'(g).$$

1. Let χ be the character of a representation of G . Prove that the inner product $(\chi, \chi) = 1$ if and only if the representation is irreducible.
2. Find the character of the i -th exterior power of the tautological representation of S_n .
3. Determine the character table for D_6 , S_4 and A_5 .
4. Determine all finite groups that have a faithful real 2-dimensional representation.
5. Let ρ be a representation of G . Prove or disprove: If the only G -invariant operators on V are multiplication by a scalar, then ρ is irreducible.
6. Prove that G has a faithful representation on a finite-dimensional complex vector space.