

Algebra I: Homework assignment 4

Due date: September 29

1. Find the automorphism group of the group $\mathbb{Z}/n\mathbb{Z}$ for any positive integer n .
2. Find a non-inner automorphism of the group A_5 .

3. Prove that any automorphism of a group G permutes conjugacy classes of G . Prove also that under this permutation any finite conjugacy class goes to a conjugacy class with the same number of elements.

4. (a) Prove that all automorphisms of S_n are inner unless $n = 6$.

Hint: Compute the number of elements in the conjugacy class of transpositions. Show that any other conjugacy class of an order 2 permutation has different number of elements (unless $n = 6$). Then prove that there are only $n!$ automorphisms that send transpositions to transpositions.

- (b) Find the outer isomorphism group of S_6 .

5. (a) Let p be a prime number. Classify all groups of order p .

(b) Classify all groups of order 6.

(c) Let p and q be different prime numbers. Classify all Abelian groups of order pq .

Bonus (d*) Let p and q be different prime numbers. Classify all groups of order pq .

6. Find all finite groups that have exactly two conjugacy classes.