

Algebra I: Homework assignment 2

Due date: September 15

1. Find the cycle decomposition of the following permutation from S_9 :

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 8 & 5 & 9 & 7 & 4 & 1 & 2 & 6 \end{pmatrix}$$

What is the order of σ ?

2. (a) Prove that the group of symmetries of an equilateral triangle is isomorphic to S_3 .
(b) Prove that the group of rotations of a tetrahedron is isomorphic to A_4 .
(c) Prove that the group of rotations of a cube is isomorphic to S_4 .
(d) For each pair of parallel faces of a cube consider the line passing through the centers of the faces. Using that all rotations of a cube permute these lines, construct an epimorphism $S_4 \rightarrow S_3$.
3. (a) Prove that a subgroup of index 2 is always normal.
(b) **Generalization.** Let p be the smallest prime divisor of the order of a group G . Prove that any subgroup in G of index p is normal.
4. For each pair of positive integers m and n find all homomorphisms from $\mathbb{Z}/n\mathbb{Z}$ to $\mathbb{Z}/m\mathbb{Z}$.
5. Prove that the symmetric group S_n is generated by two elements: the cycle $\sigma = (1\ 2\ \dots\ n)$ and the transposition $\tau = (1\ 2)$
6. Let A and B be two normal subgroups of a group G such that $A \cap B = \{1\}$. Prove that $ab = ba$ for any $a \in A$ and $b \in B$.