Applied Algebra: Homework assignment 9
Due date: November 17

1. Find the sign of the following permutations
(i)

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 9 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 11 & 12 & 10 \end{pmatrix},$$

(ii)

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 12 & 7 & 2 & 8 & 4 & 6 & 3 & 9 & 5 & 1 & 11 & 10 \end{pmatrix}$$

2. Give an example of a permutation of order 2 that is not a transposition.

3. Show that if π and σ are permutations such that $(\pi\sigma)^2 = \pi^2\sigma^2$ then $\pi\sigma = \sigma\pi$.

4. What is the highest possible order of an element in
(i) S_8 (ii) S_{12} (iii) S_{15}
Remark: There is no formula known for the highest order of an element in S_n.

5. Show that every element of S_n $(n \geq 2)$ is a product of transpositions of the form $(k\ k{+}1)$.
Hint: $(k\ k+2) = (k\ k+1)(k+1\ k+2)(k\ k+1)$

Bonus 6. In the well-known fifteen game (see the link on the course webpage to play this game), there are 15 numbered tiles inside a 4×4 board. A move is made by sliding a tile into the empty position (you can not lift tiles). Prove that it is impossible to go from arrangement A to arrangement B.

$A =$

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

$B =$

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Hint: You can think of each move as a transposition (k 16) from S_{16}*, where* 16 *represents the empty position. Then any sequence of moves is a product of transpositions. Show that if a product of such transpositions takes 16 to 16, then the number of transpositions must be even (look at the path made by the empty position on the board). Compare now the signs of permutations A and B.*