Applied Algebra: Homework assignment 8
Due date: November 10

Symmetric groups:
Let π_1, π_2, π_3, π_4 and π_5 be the following permutations from S_{12}:

$$\pi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 3 & 2 & 1 & 6 & 5 & 4 & 9 & 8 & 7 & 10 & 11 & 12 \end{pmatrix},$$

$$\pi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 10 & 11 & 12 \end{pmatrix},$$

$$\pi_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 4 & 5 & 6 & 7 & 8 & 9 & 1 & 2 & 3 & 10 & 11 & 12 \end{pmatrix},$$

$$\pi_4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 9 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 11 & 12 & 10 \end{pmatrix},$$

$$\pi_5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 12 & 7 & 2 & 8 & 4 & 6 & 3 & 9 & 5 & 1 & 11 & 10 \end{pmatrix}.$$

1. Calculate the following products: $\pi_1\pi_2$, $\pi_2\pi_3$, $\pi_3\pi_1$, $\pi_3\pi_2$, $\pi_2\pi_1\pi_3$, π_2^3, $\pi_4\pi_5$, $\pi_5\pi_4$, $\pi_1\pi_3$, π_2^2, $\pi_2\pi_1$, π_3^2, $\pi_2\pi_1\pi_2$, $\pi_2\pi_3\pi_2$, π_4^2, π_5^2.

2. Find the inverses of π_1, π_2, π_3, π_4 and π_5.

3. Write each of the permutations π_1, π_2, π_3, π_4 and π_5 as a product of disjoint cycles.

4. Find the order of π_1, π_2, π_3, π_4 and π_5.

Bonus 5. There is a deck of playing cards. You can shuffle the deck in two different ways. First, you can move the top card to the bottom. Second, you can switch two top cards (the top one and the one immediately below it). Show that using only these two shuffles you can get any permutation of the deck.

Algebraic formulation: *the group S_n is generated by the cycle* $(123...n)$ *and the transposition* $(n-1\ n)$.