

Applied Algebra: Homework assignment 6
Due date: October 13

Sets:

1. Which among the following sets are equal to one another?

$$X = \{x \in \mathbb{Z} : x^3 = x\};$$

$$Y = \{x \in \mathbb{Z} : x^2 = x\};$$

$$Z = \{x \in \mathbb{Z} : x^2 \leq 2\};$$

$$W = \{0, 1, -1\}$$

$$V = \{1, 0\}$$

2. (a) Let X be any set. Check that the empty set is a subset of X .

(b) List all subsets of the set $X = \{a, b, c\}$. How many are there?

(c) Do the same for $X = \{a, b, c, d\}$.

(d) Now suppose that the set X has n elements. How many subsets does X have?

3. Let A , B and C be any sets. Prove that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

4. Let A , B and C be any sets. Are the following true? (In each case give a proof or a counterexample.)

(a) $(A \times C) \cap (B \times D) = (A \cap B) \times (C \cap D)$

(b) $(A \times C) \cup (B \times D) = (A \cup B) \times (C \cup D)$

Euler's inclusion formula:

5. Of a group of 23 students, 10 take Linear Algebra, 11 take Geometry, 12 take Vector Calculus and 3 take none of these courses. 5 take both Linear Algebra and Vector Calculus, 6 take Vector Calculus and Geometry, and 4 take Linear Algebra and Geometry.

(a) How many students are taking all three courses?

(b) How many students are taking only one of these courses?

Functions:

6. Describe all functions from the set $X = \{0, 1, 2\}$ to the set $Y = \{0, 5\}$.