

Applied Algebra: Homework assignment 4
Due date: September 29

Linear equations:

1. Find all solutions (when there are any) of the following linear congruences:

(a) $7x \equiv 9 \pmod{13}$

(b) $8x \equiv 7 \pmod{22}$

(c) $9x \equiv 21 \pmod{12}$

2. A congruence class $[x]$ modulo n is called “identity-like” if there exists a nonzero congruence class $[y]$ modulo n such that

$$[x][y] = [y].$$

(a) Find all “identity-like” congruence classes modulo 24.

(b) Prove that a congruence class x modulo n is “identity-like” if and only if the congruence class $[x - 1]$ modulo n is a zero divisor.

Chinese Remainder Theorem:

3. Find the smallest positive integer whose remainder when divided by 9 is 4 and whose last digit is 5.

4. Solve Problem 5 from Section 1.6 of the textbook.

Polynomial equations:

5. (a) Show that the polynomial $x^4 + x^2 + 1$ has no integer roots, but that it has a root modulo 3, and factorize it over \mathbb{Z}_3 .

(b) Show that the equation $7x^3 - 6x^2 + 2x - 1 = 0$ has no integer solutions.

6. Find all solutions of the equation $x^4 - 1 = 0$ modulo n , where

(a) $n = 5$ (b) $n = 6$ (c) $n = 7$ (d) $n = 8$