**Applied Algebra: Homework assignment 3**
**Due date: September 22**

**Congruence classes:**

**1.** Show that no integer of the form $8n + 7$ can be written as a sum of three squares.

**2.** Let $p$ be a prime number. Show that the equation $x^2 = [1]_p$ has just two solutions in $\mathbb{Z}_p$. Give a counterexample to this statement when $p$ is not prime.

**3.** Compute the sum and the product of the congruence classes $a$ and $b$ modulo 17 for
**(a)** $a = [16]$, $b = [14]$;
**(b)** $a = [5]$, $b = [13]$;
**(c)** $a = [8]$, $b = [7]$.

**4.** Find the following inverses if they exist:
**(a)** the inverse of 7 modulo 11;
**(b)** the inverse of 10 modulo 26;
**(c)** the inverse of 11 modulo 31.

**5. (a)** Find the last digit of the number $17^{35}$.
**(b)** Find the last two digits of the same number.

**Bonus 6.** Prove that an integer $p$ is prime if and only if

$$(p-1)! \equiv -1 \pmod{p}.$$

Recall that $n! = 1 \cdot 2 \cdot \ldots \cdot n$.