Applied Algebra: Homework assignment 11
Due date: December 6

Cosets and Lagrange's theorem

1. (a) Let $G = G_{14}$ be the group of all invertible congruence classes modulo 14. Write down the distinct left cosets of the subgroup $\{[1]_{14}, [13]_{14}\}$.

(b) Let $G = S_3$ be the symmetric group, and H the Klein four group. Write down the dictinct left cosets of H.

2. Show that $\varphi(n)$ is divisible by 2 for any $n \geq 3$.
[Hint: note that $(-1)^2$=1.]

Error-correcting codes:

3. (a) For each of the following generator matrices, say how many errors the corresponding code detects and how many errors it corrects:

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}; \quad \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix};$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}; \quad \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

(b) Which of the codes of part (a) is the most efficient?

4. Let $f : \mathbf{B}^3 \to \mathbf{B}^9$ be the coding function given by

$$f(abc) = abcabc\bar{a}\bar{b}\bar{c},$$

where $\bar{x}$ is 1 if x is 0 and $\bar{x}$ is 0 if x is 1. List the eight codewords of f. Show that f does not give a group code. How many errors does f detect and how many errors does it correct?

Bonus 5. There are two identical large boxes filled with oranges. In both boxes, oranges are arranged into horizontal layers so that each layer is as close to the layer below as possible. In the first box, the oranges in each layer form the hexagonal (honeycomb) pattern. In the second box, they form the square-like pattern. Which box contains more oranges? (Boxes are so large that you can ignore disruptions in the pattern near the boundaries. Oranges are supposed to be perfect spheres of equal size.)