

**Applied Algebra: Homework assignment 10**  
**Due date: November 29**

1. Decide which of the following sets are groups under the given operations. Explain your answer.

- (i) the set of rational numbers  $\mathbb{Q}$ , under multiplication;
- (ii) the non-zero integers under multiplication;
- (iii) the set of all functions from  $\{1, 2, 3\}$  to itself under composition of functions
- (iv) the set of integers under subtraction
- (v) permutations  $e, (12)(34), (13)(24), (14)(23)$ , under composition
- (vi) the set of real numbers under the operation  $*$  defined by  $a * b = a + b - 2$

2. Construct an isomorphism between the group of Problem 1(vi) and the group of real numbers, under addition.

3. Let  $G = (\mathbb{Z}/8\mathbb{Z})^*$  be the group of all invertible congruence classes modulo 8.

- (a) Find the order of  $G$ .
- (b) Find all elements of order 2 in  $G$ .
- (c) Construct an isomorphism between  $G$  and the **Klein four group** of Problem 1(v).

4. Prove that an inverse of any element of a group is unique.

5. Let  $G$  be a commutative group, and  $g, h \in G$  elements of orders 2 and 3, respectively. Find the order of the elements  $g^2, h^2, gh$  and  $gh^2$ .

**Bonus 6.** Let  $G$  be a group in which  $a^2 = e$  for all elements  $a \in G$ . Show that  $G$  is Abelian.