

Applied Algebra: Homework assignment 1
Due date: September 8

Euclidean algorithm:

1. For each of the following pairs m and n of integers, find the greatest common divisor d of m and n and find integers x and y such that $d = mx + ny$:

(a) $n = 20$, $m = 13$;

(b) $n = 126$, $m = 91$;

(c) $n = 77695236973$, $m = 6003722857$. [The Euclidean algorithm requires only 3 steps for these integers.]

2. Suppose you have infinitely many 3-dollar and 5-dollar bills.

(a) How to pay 101 dollars without change using these bills?

(b) Show that using these bills you can pay without change any amount greater than 7 dollars.

3. Let a , b and c be integers.

(a) Show that if a and c are relatively prime, and b and c are relatively prime, then ab and c are also relatively prime.

(b) Is it true that if c does not divide a , and c does not divide b , then c does not divide ab ? Justify your answer.

Bonus 4. Using that

$$718865222040754575648532881408$$

is equal to x^{13} for some integer x , find x without calculator.

Hint: use criteria for divisibility by 2, 9 and 11.