My NAME is:

Problem	1	2	3	Total
Score				

MAT 342
Applied Complex Variables
Midterm 1

February 27, 2007

CALCULATOR AND CELLPHONE POLICY: No calculators or computers may be used on this text. NO CELLPHONES are permitted in the examination room.

Show all your work on these pages! Total score = 100

1. (a) (15 points) What are the 4 fourth roots of -9?

(b) (15 points) Write $z^4 + 9$ as $(z - r_1)(z - r_2)(z - r_3)(z - r_4)$.

(c) (15 points) Use the fact that the complex roots of a polynomial with real coefficients come in *complex conjugate pairs* to write $z^4 + 9$ as a product of two quadratic polynomials with real coefficients.

2. (a) (15 points) What is the image of the line $\Im(z) = 1$ [i.e. $\{x + iy | y = 1\}$] under the mapping $w = z^2$?

(b) (15 points) Sketch the image of the half-plane $\Im(z) \geq 1$ under the mapping $w = z^2$.

3. (a) (15 points) Show carefully by an ϵ, δ argument that

$$\lim_{z \to a} \frac{f(z)}{g(z)} = 0$$

if $\lim_{z \to a} f(z) = 0$ and if there exists a pair of positive numbers δ_0, M such that $|z - a| < \delta_0$ implies $|g(z)| \geq M$.

(b) (10 points)Apply this to prove that

$$\lim_{z \to 0} \frac{z}{2 + \frac{\overline{z}}{z}} = 0.$$