My NAME is:

Problem	1	2	3	4	Total
Score					

MAT 342
Applied Complex Analysis
Midterm 2

April 12, 2007

Show all your work on these pages! Total score = 100

You may quote theorems we have learned this semester, but for full credit you must state them carefully, check that the hypotheses are satisfied, and (of course) use them appropriately.

1. (a) (12 points) Using the definition $e^z = e^x \cos y + ie^x \sin y$, where $z = x + iy$, show that the function $f(z) = e^z$ is analytic.

(b) (12 points) Taking $e^z = e^x \cos y + ie^x \sin y$ as your definition, show that

$$\frac{d}{dz} e^z = e^z.$$

2. (a) (14 points) Evaluate

$$\int_C \frac{dz}{z^2+2z+4}$$

where C is the circle of radius 2 about $2i$, traversed counterclockwise.

(b) (12 points) Show that if C_R is the semicircle $|z| = R$, $\Im(z) \geq 0$ ($\Im(z)$ is the imaginary part of z), then

$$\lim_{R\to\infty} \int_{C_R} \frac{dz}{z^2+2z+4} = 0.$$

Hint: you may want to use the triangle inequality in the form $|a+b+c| \geq |a| - |b| - |c|$.

(c) (14 points) Calculate $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+4}$. (If you can do this without complex analysis, that's fine too).

3. (12 points) Evaluate $\int_C \frac{e^{3z}}{z^2}\, dz$, where C is the circle $|z| = 1$, traversed counterclockwise.

4. (a) (12 points) Show that

$$\int_C \frac{dz}{z^4} = 0,$$

where C is the circle $|z| = 1$, traversed counterclockwise, by direct calculation or by quoting an *appropriate* theorem.

(b) (12 points) Show that

$$\int_S \frac{dz}{z^4} = 0,$$

where S is the boundary of a pentagon with vertices at $3i, \pm 3, \pm 2 - 2i$, by a method of your choice.