MAT 312 Spring 2009 Homework 12

1. Find $q(x)$ and $r(x)$ in $\mathbf{Z}(X)$ to satisfy

$$x^4 - 5x^3 + 3x^2 + x = (x^2 + 1)q(x) + r(x)$$

with degree$(r) < 2$.

2. Prove that in $\mathbf{Z_2}(X)$ the polynomial $x^2 + x + 1$ is irreducible (cannot be factored as a product of two linear polynomials).

3. In $\mathbf{Z}(X)$ calculate the greatest common divisor $d(x)$ of $a(x) = x^4 - 5x^3 + 3x^2 - 5x + 2$ and $b(x) = x^5 + x^4 + 2x^3 + 2x^2 + x + 1$. Then use the Euclidean algorithm to find polynomials $h(x)$ and $k(x)$ such that $d(x) = h(x)a(x) + k(x)b(x)$.

4. Let $\omega = e^{\pi i/16}$. Then ω is a primitive 32-nd root of 1. Show how the 32 factors

$$(x - 1)(x - \omega)(x - \omega^2) \cdots (x - \omega^{31})$$

can be grouped two by two to give a product of 16 quadratic factors; show that these can be grouped two by two to give a product of 8 degree-4 factors; continue through three more steps to get $x^{32} - 1$.