Section 9.3

3)

$$\Omega = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$$

a) Applying Ω to $\begin{pmatrix} 1 \\ 2 \\ 0 \\ 2 \end{pmatrix}$ gives $\begin{pmatrix} 5 \\ 1 \\ -3 \\ 1 \end{pmatrix}$.

b) Applying Ω to $\begin{pmatrix} i \\ -1 \\ 2i \\ 2 \end{pmatrix}$ gives $\begin{pmatrix} 1+3i \\ -4i \\ -1+3i \\ 2i \end{pmatrix}$.

Additional Exercises

$$\Omega = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 & -1 & -\omega & -\omega^2 & -\omega^3 \\ 1 & \omega^2 & -1 & -\omega^2 & 1 & \omega^2 & -1 & -\omega^2 \\ 1 & \omega^3 & -\omega^2 & \omega & -1 & -\omega^3 & \omega^2 & -\omega \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\omega & \omega^2 & -\omega^3 & -1 & \omega & -\omega^2 & \omega^3 \\ 1 & -\omega^2 & -1 & \omega^2 & 1 & -\omega^2 & -1 & \omega^2 \\ 1 & -\omega^3 & -\omega^2 & -\omega & -1 & \omega^3 & \omega^2 & \omega \end{pmatrix}$$

1) Applying this matrix to $f = (-1, -1, 1, 1, 1, 1, -1, -1)$ yields

$$\begin{pmatrix} 0 \\ -2-2\sqrt{2}+2i \\ 0 \\ -2+2\sqrt{2}-2i \\ 0 \\ -2+2\sqrt{2}+2i \\ 0 \\ -2-2\sqrt{2}-2i \end{pmatrix}$$

Taking norms gives:

$$\begin{pmatrix} 0 \\ 2\sqrt{4+2\sqrt{2}} \\ 0 \\ 2\sqrt{4-2\sqrt{2}} \\ 0 \\ 2\sqrt{4-2\sqrt{2}} \\ 0 \\ 2\sqrt{4+2\sqrt{2}} \end{pmatrix}$$

2) Applying this matrix to $f = (-1, -1, 1, 1, -1, -1, 1, 1)$ yields

$$\begin{pmatrix} 0 \\ 0 \\ 4+4i \\ 0 \\ 0 \\ 0 \\ 4-4i \\ 0 \end{pmatrix}$$

Taking norms gives:

$$\begin{pmatrix} 0 \\ 0 \\ 4\sqrt{2} \\ 0 \\ 0 \\ 0 \\ 4\sqrt{2} \\ 0 \end{pmatrix}$$

3) Applying this matrix to $f = (-1, 1, -1, 1, -1, 1, -1, 1)$ yields

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -8 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Taking norms gives:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 8 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The answers above were achieved using the formula for the DFT in the book, which is simply matrix multiplication of Ω with f. If you used the definition of the DFT that I am told was given in class, $\frac{1}{N}\Omega f$), then you would arrive at the answers above, all divided by 8.