MAT 312/AMS 351 – Fall 2010
Homework 12b

1. Is this a group code? Explain your answer.

$$\begin{array}{|ccc|}\hline 0&0&0\\ 0&0&1\\ 0&1&0\\ 0&1&1\\ 1&0&0\\ 1&0&1\\ 1&1&0\\ 1&1&1\\ \hline\end{array} \to \begin{array}{|ccc|c|}\hline 0&0&0&1\\ 0&0&1&0\\ 0&1&0&1\\ 0&1&1&0\\ 1&0&0&1\\ 1&0&1&0\\ 1&1&0&1\\ 1&1&1&0\\ \hline\end{array}.$$

2.

$$f : \begin{array}{|ccc|}\hline 0&0&0\\ 0&0&1\\ 0&1&0\\ 0&1&1\\ 1&0&0\\ 1&0&1\\ 1&1&0\\ 1&1&1\\ \hline\end{array} \to \begin{array}{|ccc|ccc|}\hline 0&0&0&0&0&0\\ 0&0&1&0&1&1\\ 0&1&0&1&0&1\\ 0&1&1&1&1&0\\ 1&0&0&1&1&0\\ 1&0&1&1&0&1\\ 1&1&0&0&1&1\\ 1&1&1&0&0&0\\ \hline\end{array}$$

and

$$g : \begin{array}{|ccc|}\hline 0&0&0\\ 0&0&1\\ 0&1&0\\ 0&1&1\\ 1&0&0\\ 1&0&1\\ 1&1&0\\ 1&1&1\\ \hline\end{array} \to \begin{array}{|ccc|cccc|}\hline 0&0&0&0&0&0&0\\ 0&0&1&0&1&1&1\\ 0&1&0&1&0&1&1\\ 0&1&1&1&1&0&0\\ 1&0&0&1&1&0&1\\ 1&0&1&1&0&1&0\\ 1&1&0&0&1&1&0\\ 1&1&1&0&0&0&1\\ \hline\end{array}$$

are both group codes. Calculate the minimum distance d between code-words for f and for g. Is g better than f for error-detection? Is g better than f for error-correction?

3. In the previous exercise, f can be used to detect up to 2 errors in the transmission of a word, or to correct 1 error in the transmission of a word, but not both. Explain carefully why.

4. Suppose the code f above is modified to a code g' by using two copies of the third check bit:

$$g' : \begin{array}{|ccc|}\hline 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ \hline \end{array} \to \begin{array}{|ccc|cccc|}\hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline \end{array}.$$

Is g' a group code? Is g' any better than f for error-detection and correction? Explain.

5. Write out explicitly (as above) the code $f : \mathbf{B}^4 \to \mathbf{B}^7$ with generator matrix:

$$\left(\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right).$$

What is d for this code? How could you make it more useful for error control without increasing code-word length?