MAT 310 Spring 2008 Review for Final Exam

Theorems from *Axler, 2nd Ed.* you should know how to use **and how to prove**.

2.18 $\dim(U_1 + U_2) = \dim U_1 + \dim U_2 - \dim(U_1 \cap U_2)$.

3.17 $T : V \to U$ is invertible $\Leftrightarrow$ T injective and surjective.

3.18 Two finite-dimensional vector spaces over the same field $\mathbf{F}$ are isomorphic $\Leftrightarrow$ they have the same dimension.

5.10 V finite-dimensional over $\mathbf{C}$: every operator $T : V \to V$ has an eigenvalue. Assume Fundamental Theorem of Algebra!

5.13 V finite-dimensional over $\mathbf{C}$, $T : V \to V$, V has a basis $\mathcal{B}$ such that the matrix $[T]_{\mathcal{B}}$ is upper-triangular.

5.18 If $[T]_{\mathcal{B}}$ is upper-triangular, the diagonal elements of $[T]_{\mathcal{B}}$ are the eigenvalues of T.

6.6 Cauchy-Schwarz inequality.

6.9 Triangle inequality.

6.20 Gram-Schmidt orthonormalization.

6.28 Every finite-dimensional inner-product space has an orthonormal basis.

6.xx V finite-dimensional over $\mathbf{C}$, $T : V \to V$, V has an *orthonormal* basis $\mathcal{B}$ such that the matrix $[T]_{\mathcal{B}}$ is upper-triangular.

6.33 $(U^\perp)^\perp = U$

6.45 V a finite-dimensional inner-product space. Every $\varphi : v \to \mathbf{F}$ is of the form $\varphi(v) = \langle v, w_\varphi \rangle$.

6.47 If $\mathcal{B}$ is an orthonormal basis, and T^* is the adjoint of T, $[T^*]_{\mathcal{B}} = \overline{[T]_{\mathcal{B}}^t}$ the complex conjugate transpose.

6.46 null $T^* = (\text{range } T)^\perp$, etc.

7.1 Every eigenvalue of a self-adjoint operator is real.

7.2 If $T : V \to V$, V a $\mathbf{C}$-vector space, and $\langle Tv, v\rangle = 0$ for every $v \in V$, then $T = 0$.

7.6 T is normal $\Leftrightarrow ||Tv|| = ||T^*v||$ for every $v \in V$.

7.9 $\mathbf{C}$-Spectral Theorem.

7.36ab (Characterization of isometry): T preserves norms $\Leftrightarrow$ T preserves inner products.

7.37 $T : V \to V$, V a $\mathbf{C}$-vector space. T is an isometry $\Leftrightarrow$ V has an orthonormal basis of T-eigenvectors with associated eigenvalues all having absolute value equal to one.

8.5 $T : V \to V$. If null $T^m =$ null T^{m+1} then null $T^{m+1} =$ null $T^{m+2} =$ etc.

8.6 $T : V \to V$, $\dim V = n$. null $T^n =$ null T^{n+1}.

8.20 Cayley-Hamilton Theorem for operators on $\mathbf{C}$-vector spaces.

8.23 $T : V \to V$, V a $\mathbf{C}$-vector space. Let $\lambda_1, \ldots, \lambda_m$ be the *distinct* eigenvalues, and $U_1, \ldots, U_m$ the associated generalized eigenspaces. Then $V = U_1 \oplus \cdots \oplus U_m$, the subspace U_j is T-invariant for $j = 1, \ldots, m$ and $(T - \lambda_j I)|_{U_j}$ is nilpotent.

8.26 If $T : V \to V$ is nilpotent, then V has a basis $\mathcal{B}$ such that $[T]_{\mathcal{B}}$ has only zeros on or below the diagonal.