My NAME is: in Section .

Problem	1	2	3	Total
Score				

MAT 310
Linear Algebra
Midterm 2

November 17, 2008

No books or notes may be consulted during this test.
Explain your answers carefully. Show all your work in the "yellow book."
Total score = 100.

1. We know (*) that any invertible matrix can be written as a product of elementary matrices.

   (a) (10 points) Show that
   $$A = \begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$$
   is invertible. Explain carefully.

   (b) (20 points) Express A as a product of elementary matrices.

   (c) (15 points) Suppose E is an elementary $n \times n$ matrix of type 1, F is an elementary $n \times n$ matrix of type 2, G is an elementary $n \times n$ matrix of type 3 and M is an arbitrary invertible $n \times n$ matrix. How are the determinants $\det EM$, $\det FM$ and $\det GM$ related to $\det M$? You may use only the following properties of the determinant:

   - If all the rows of a matrix M are fixed except one, $\det(M)$ is linear as a function of the variable row.
   - If a multiple of one row is added to another row, det does not change.
   - If two rows are interchanged, det changes sign.

2. (15 points) Calculate the determinant
$$\det \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 \\ 5 & 4 & 3 & 2 \end{bmatrix}.$$

3. Let $\mathcal{P}_2$ represent the vector space of polynomials $p(x) = a_0 + a_1 x + a_2 x^2$, with a_0, a_1, a_2 real numbers, and let $L\colon \mathcal{P}_2 \to \mathcal{P}_2$ be defined by $L(p)(x) = xp'(x) + p(x)$, where p' is the derivative of p.

   (a) (20 points) Prove carefully that L is a linear transformation.

   (b) (10 points) What is the matrix $[L]_\beta^\beta$ of L with respect to the basis $\beta = (1, x, x^2)$ for $\mathcal{P}_2$?

   (c) (10 points) $\mathcal{P}_2$ also has the basis $\gamma = (x + x^2, 1 + x^2, 1 + x)$. Show that $[L]_\beta^\beta$ and $[L]_\gamma^\gamma$ must have the same determinant. You may use the properties of determinant mentioned above, along with $\det(AB) = \det(A)\det(B)$.

END OF EXAMINATION