MAT125 Fall 2013 Final Review

All material in Review 1 and Review 2 plus the following.

3.3 Be able to sketch the graphs of $\sin x$ and $\cos x$ to scale ($\pi = 3.14..$) and to convince yourself that $\sin' = \cos$ and that $\cos' = -\sin$. (Boxes 4, 5 pp.193; Example 1 p.193; Exercises 1, 4). Remember or be able to calculate that $\tan' = \sec^2$. [This is a good place to review your trigonometric identities, especially $\sin^2 + \cos^2 = 1$ and $\tan^2 + 1 = \sec^2$.] Example 2; Exercises 3, 5, 6, 8

3.4 Know and be comfortable with the Chain Rule in both forms (Box, p.198). Example 1, p.199: read the Note at the end. Practice as many examples as you can. Special case with the "outside function" a power: Box 4 on p.200; Examples 3, 4, 5. Be able to apply the Chain Rule to $f(x) = a^x = e^{x \ln a}$ to obtain $f'(x) = a^x \ln a$ (Box 5, p.202; Exercise 22).

3.5 Implicit differentiation always involves the chain rule. In a case like Example 1 p.210, where we are differentiating with respect to x, the derivative of x^2 is $2x$ but the derivative of y^2 is $2y\frac{dy}{dx}$. The pattern is always the same: take $\frac{d}{dx}$ of *everything*, then solve for $\frac{dy}{dx}$ (Example 2; Exercises 3, 4, 5).

3.6 Be able to use implicit differentiation to calculate the derivatives of the inverse trigonometric $\sin^{-1}$ and $\tan^{-1}$ (Boxes, p.217, 219). (Remember they are aslo called arcsin and arctan). [Need those trig identities.] Exercises 27-30.

3.7 Understand why $\frac{d}{dx} \ln x = \frac{1}{x}$, and be able to use this fact with the Chain Rule as in Examples 1, 2, 3 on p.222 (Exercises 2, 3, 8). Know how and when to apply *logarithmic differentiation* (Box, p.224; Example 8; Exercises 33, 38).

3.9 Linear approximation. Be able to use the value $f(a)$ and the derivative $f'(a)$ to *estimate* $f(x)$ for x near a. Equation 1 p. 241, Example 1, Exercises 15, 17. Understand that if the graph of f is concave up at a, then linear approximation will underestimate nearby $f(x)$; and overestimate if the graph is concave down (Example 2). Be able to use linear approximation (*differentials*) to estimate how errors in measurement propagate in calculations: Example 4, Exercises 27, 28, 29.

4.1 Related rates. Work through Examples 1, 2, 3, 4, 5 understanding how they reflect the "Strategy" explained on p.258. Note that the solution will always involve the Chain Rule. Exercises 4, 10, 17 and as many others as you can fit in.

4.2 Understand the definitions (Box 1, p.262) of "f has an absolute maximum at c" and "$f(c)$ is the maximum value of f" (and for "minimum" also). [In particular understand the distinction in mathematical usage between *a maximum* and *a maximum value*: $f(x) = 1 - x^2$ has its maximum at $x = 0$. Its maximum value is $f(0) = 1$.] Understand the definition (Box 2, p.263) of "f has a local maximum at c," etc. and study Example 4 carefully. Understand what "Fermat's Theorem" says (Box 4, p.265) and the situations of Figure 11 (you can have $f'(c) = 0$ without c being a local max or min) and Figure 12 (f might not have a derivative at the point c where $f(c)$ is minimum or maximum). Be able to implement the "Closed

Interval Method" (Box, p.266): Example 6b, Exercises 5, 41, 42, 52.

4.3 Understand that if $f'(x) > 0$ on an interval, then f is "sloping up" and therefore increasing on that interval; and that if $f'(x) < 0$ on an interval, then f is "sloping down" and therefore decreasing on that interval (Box, p.273), Example 2, Exercises 7a, 8a, 9a. Understand the relation between f'' and the concavity of the graph of f (Box, p.275): remember Paul Kumpel's mnemonograms:

$$\begin{array}{c}+\;\;+\\ \smile\end{array}\qquad\qquad\begin{array}{c}-\;\;-\\ \frown\end{array}$$

Understand what an *inflection point* is (just below Figure 5 on p.275). Exercises 7c, 8c, 9c. The Second Derivative Test (Box, p.275) will be useful when you get to optimization problems, in section 4.6. Be able to put slope and concavity information together with information about asymptotes (from section 2.5) to sketch graphs of complicated functions. Example 4, Exercises 33, 38.

4.5 Understand *when* you can apply L'Hospital's rule: BOTH numerator and denominator must have limit 0, or BOTH numerator and denominator must have ∞ limit (can be plus or minus ∞). Box, p.291, Example 1. Be prepared to apply the rule more than once, Example 2. Exercises 5, 9, 17. Be able to rewrite an *indeterminate product* (p.294, i.e. where one factor goes to 0 and the other goes to ∞ or to $-\infty$) as a quotient suitable for L'Hospital's rule. Example 6, Exercises 29, 30. And *indeterminate differences* Example 7, Exercises 33, 35. And *indeterminate powers*: Examples 8 and 9, Exercises 39, 40.

4.6 Optimization problems are difficult because (as in related rates problems) you have to set up the notation and the mathematical context. Understand and practice applying the "Six-step process" from p.299-300. Steps 4 and 5 are crucial: this is where you make the word problem into a "find the maximum" (or minimum) problem. Work through Examples 1, 2, 3, 4, 5 *with the six steps in mind*, so you learn how to apply them yourself. Exercises 15, 19, 22 and as many others as you can manage. Do odd-numbered ones so you can check your answers in the back of the book.

Use the Chapter Reviews for further reviewing.

November 28, 2013