MAT125 Fall 2013 Review for Midterm II

revised, October 23 2013

2.2 Understand that the limit of a function $f(x)$ at $x = a$ does not involve the value $f(a)$ (which may not even be defined). (Definition, p.95) (Example 1). Also definitions of left- and right-hand limits (Box, p.100 and below). Be able to estimate $\lim_{x\to a} f(x)$, as well as one-sided limits, from inspection of the graph of f (Exercises 4,5).

2.3 Understand that limits interchange with the arithmetic operations $+, \times, -, \div$ except when they lead to division by zero (Example 1, Exercise 2,3,8). In particular since $\lim_{x\to a} c = c$ (here c is a constant function) and $\lim_{x\to a} x = a$ (be sure you understand what this means), the limit at $x = a$ of a polynomial function of x is the value of that function at a (Example 2a). This does not always work for *quotients* of polynomials (Examples 2b, 3, Exercises 12, 13, 14).

2.4 Know the definition of "f continuous at a" (Box, p.113) (Exercises 3a,13,14). Also "continuous from the right" and "from the left" (Box 2, p.115) (Exercise 3b). Understand that polynomials are continuous everywhere, and that rational functions are continuous wherever they are defined (Theorem 5, p.116) (Example 5). Be able to apply the Theorem on compositions (Theorem 8 p.119) (Examples 8,9). Understand how to use the Intermediate Value Theorem to calculate roots of equations by repeated approximation (Example 9, Exercise 45).

2.5 Understand how $\lim_{x\to a} f(x) = \infty$, etc., give a *vertical asymptote* at a (Box 2, p.125, Example 1). Exercises 3, 7; and how $\lim_{x\to\infty} f(x) = L$, etc., give as *horizontal asymptote* the line $y = L$ (Box 5 p.128, Examples 3,4 p.129). Exercises 3, 7.

Be able to calculate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for the special case where $f(x) = p_1(x)/p_2(x)$ is the quotient of two polynomials, p_1, p_2. Use the "divide by the highest power of x in the denominator" method (explained on p.130). Example 5 p.130, Example 10 p.132. Exercises 16, 20, 22.

2.6 Understand that the slope of the tangent line is the limit of the slopes of secant lines (Figure 1, Box 1, p.135). Be comfortable with both notations: $x \to a$ and $h \to 0$, where $h = x - a$. (Compare Box 4 and Box 5, p.138). Be able to write the equation of the tangent line to the graph of f at the point $(a, f(a))$ using $f'(a)$ and the point-slope formula. Examples 1,2 p.136. Exercises 7,8. Understand also that instantaneous velocity at $t = a$ is the limit of average velocities over smaller and smaller time periods beginning or ending with a. Box 3 p.137. Example 3. Exercises 15, 16ab.

2.7 Basic concept: the derivative of f at x is a new function $f'(x)$ (Box 2 p.146). Be able to estimate (and sketch) f' when f is given by a graph. Example 1. Be able to estimate (and sketch) f' when$f$ is given by a table. Example 2. Be able to calculate f' *from the definition* when f is given by formula, in certain simple cases: Examples 3, 4, 5. Be familiar with the two notations: if f is a function of x, then $f'(a) = \frac{df}{dx}|_a$ (p. 150). Be able to recognize points on a graph where the function is *not* differentiable. (Figure 8, p. 152. Exercises

35-38.) Be able to calculate the second derivative f'' as the derivative of f' (Example 7 p. 153). Understand that if $f(t)$ is position as a function of time, then $f''(t)$, the the derivative of velocity, is acceleration (Example 2 p.154) and that $f'''(t)$, the derivative of acceleration, can be felt, while driving, as "jerk". Exercises 43, 44, 48.

2.8 Be able to tell by examining f' where f is increasing and where it is decreasing (Box, p.158; Example 1, Exercises 1,2). Be able to tell from f'' where the graph is concave upward and where it is concave downward (Box, p.159; Example 2, Exercise 8). Be able to sketch the graph of f given the graph of f'. (f is the *anti-derivative* of f'). Example 3 p.160, Example 4 p.161. Exercises 29,30.

3.1 Know the elementary differentiation rules: If $f(x) = c$ (c a constant) then $f'(x) = 0$. If $f(x) = x$ then $f'(x) = 1$ [Informally, $\frac{d}{dx}(c) = 0$ and $\frac{d}{dx}(x) = 1$]. (Boxes, p.174) and understand what these equations mean in terms of slopes. Know the *Power Rule*: If $f(x) = x^n$ then $f'(x) = nx^{n-1}$ [$\frac{d}{dx}(x^n) = nx^{n-1}$]. (Boxes, p.175 and p.176). Be familiar with the special cases $n = \frac{1}{2}$ ($f(x) = \sqrt{x}$) and $n = -1$ ($f(x) = \frac{1}{x}$). Examples 2, 3. Be able to calculate the derivative of $rf(x) + sg(x)$ for constants r, s knowing the derivatives of f and g separately. (Boxes, pp.177-178 Examples 4, 5 Exercises 16,18).

Know how to differentiate the "natural exponential function" $f(x) = e^x$ (Box, p.180; Example 8, Exercises 10, 29).

3.2 Be able to apply the product and quotient rules correctly (Box, p.184; Examples 1a, 2, 3; Exercise 3,4). (Box, p.187; Examples 5, 6; Exercises 5, 6, 8). If you can't remember where the minus sign goes in the quotient rule, use $\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$ to check.

Use the Chapter Reviews for further reviewing. October 23, 2013