

Topology MAT 530 Homework 2

1. Prove that an open connected subset of \mathbb{R}^n is path connected.
2. What are the connected components of the *Cantor set*, consisting of all numbers from the segment $[0, 1]$ such that the digit 1 does not appear in their base-3 expansions?
3. Prove that the Cantor set is compact.
4. Define an equivalence relation on nonzero vectors from \mathbb{R}^n as “being parallel”. The corresponding quotient space $\mathbb{R}P^n$ is called the real *projective space*. Define a metric on $\mathbb{R}P^n$ that gives the quotient topology.
5. Prove that real projective spaces are compact. *Hint:* use that the spheres are compact.
6. A *topological group* is a group G equipped with a topology such that the maps

$$\begin{aligned}G \times G &\rightarrow G, & (g, h) &\mapsto gh, \\G &\rightarrow G, & g &\mapsto g^{-1}\end{aligned}$$

are continuous. Prove that the connected component of the identity in a topological group is a normal subgroup (a subgroup $H \subset G$ is *normal* if $gHg^{-1} = H$ for every $g \in G$).

7*. A *topological n -manifold* is a topological space such that every its point has a neighborhood homeomorphic to \mathbb{R}^n . Prove that any compact connected topological 1-manifold is homeomorphic to a circle.