

PARITY

Problem 1. Eleven cogwheels are linked in a circular order (so that the first touches the second, the second touches the third, etc., the eleventh touches the first). Can the cogwheels rotate?

Problem 2. Can a line not containing vertices of a (non-convex) 11-gon intersect all its edges?

Problem 3. Can one draw a curve on the surface of a dodecahedron that intersects each edge once? (A dodecahedron is a regular polyhedron with 12 pentagonal faces).

Problem 4. Start with 4 numbers 0, 1, 0, 0. In one step, one can add one to any pair of numbers. By doing finitely many such steps, can one make all numbers equal?

Problem 5. Can one put plus or minus signs between all digits of the number 123456789 so that to make the result equal 0?

Problem 6. Seven positive integers are written on a circle. Prove that there are two adjacent numbers, whose sum is even.

Problem 7. Consider n integers. Prove that the sum of all pairwise distances between these integers is even (the distance between two numbers a and b is defined as $|a-b|$).

Problem 8. Three grasshoppers sit on the line. The left and the right grasshoppers are at distance 1 from the middle one. Any grasshopper can jump over any other grasshopper and land at the symmetric point. Suppose that, after several jumps, the grasshoppers sit at the same points, but in different order. Prove that the left one and the right one interchanged.

Problem 9. Can a chess knight make a series of moves, starting at cell a1, arriving at cell h8, and visiting each cell exactly once?

Problem 10. The numbers x_1, \dots, x_n are equal to ± 1 . Suppose that $x_1x_2 + x_2x_3 + \dots + x_{n-1}x_n + x_nx_1 = 0$. Prove that n is divisible by 4.