Math Battle I

Problem 1. Let E and V denote the number of edges and vertices, respectively, in a convex bounded 3-dimensional polyhedron. Prove that $E \leq 3V - 6$.

Problem 2. Consider three pair-wise intersecting circles. For each pair of circles, draw the line through their intersection point. Prove that these three lines are either all parallel or all pass through the same point.

Problem 3. Is there a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f(f(x)) = e^{-x}$ for all $x \in \mathbb{R}$?

Problem 4. Is there a sequence of positive integers such that any positive integer can be uniquely represented as a difference of two terms in the sequence?

Problem 5. Positive numbers a, b and c are such that $abc = 1$. Prove that

$$\frac{1}{1+a+b} + \frac{1}{1+b+c} + \frac{1}{1+a+c} \leq 1$$

Problem 6. Does the sequence $1+17n^2$, $n \in \mathbb{N}$, contain an infinite number of squares?