Diophantine equations

Problem 1. Prove that the equation

$$\frac{x}{y}+\frac{y}{z}+\frac{z}{x}=1$$

is unsolvable in positive integers.

Problem 2. Find all integers x, y such that $xy = x + y$.

Problem 3. Find positive integers x, y and z satisfying the equation

$$x+\cfrac{1}{y+\cfrac{1}{z}}=\frac{10}{7}.$$

Problem 4. Find all integers x, y such that $3x + 5y = 1$.

Problem 5. Prove that there are infinitely many triples of integers $x > 1$, $y > 1$ and $z > 1$ such that $x! = y! \cdot z!$

Problem 6. Is there a sphere in $\mathbb{R}^3$, which has only one rational point (a point is said to be *rational* if it has rational coordinates)?

Problem 7. Prove that there are infinitely many integer solutions of the equation $x^2 + y^3 = z^5$.

Problem 8. Find all integer solutions of the equation $x^2 + y^2 + z^2 = 2xyz$.

Problem 9. Prove that the following equations

(1) $x^2 + 1 = py$,
(2) $x^2 + x + 1 = py$

are solvable in integers for infinitely many primes p.