

ALGORITHMS

Problem 1. There are 10 poisons (P1, P2, etc., P10) that look and taste the same as pure water. If somebody takes any of these poisons, this person will die. However, a poison with a bigger number can serve as an antidote to a poison with a smaller number. For example, if you take P7, then you can only be saved if you also take P8, P9 or P10. Two poisoners, A and B, arranged for a duel, during which they must drink whatever the opponent offers. Nobody has an access to poison P10 except for B. So B thought it would be easy for him to win: he offers A to drink P10, and after B drinks whatever poison A gives him, he will drink P10 as an antidote. However, A survived and B died. How could this happen?

Problem 2. You have a 12-gallon can of beer, one empty 5-gallon can, and one empty 8-gallon can. How to measure exactly 3 gallons, using only the cans you have? How to measure exactly 6 gallons?

Problem 3. I guessed a number from 1 to 15. Can you determine what my number is by asking only 4 yes or no questions? What if you are only allowed to ask 3 questions?

Problem 4. I guessed n positive integers x_1, \dots, x_n . You are allowed to ask two questions of the form “What is $a_1x_1 + \dots + a_nx_n$?” for some integers a_1, \dots, a_n you choose. How to determine all integers I guessed?

Problem 5. In a group of $k \geq 3$ people (living far from each other), each learned a news that nobody else was aware of. Any two people can share their news over the phone, so that at the end of the conversation both new all news of each other. Prove that it suffices to make $2k - 4$ phone calls to share all k news between all k people.

Problem 6. Prove that using only one operation $a * b = 1 - a/b$, one can compute the sum, the product, the difference and the ratio of any two numbers.

Problem 7. How many multiplications do you need to perform to compute x^{15} ?

Problem 8. A programmable calculator can perform the following operations: $+$, $-$, \times , $/$ and $\sqrt{\cdot}$. Write a program to find the minimum of two numbers.