Calculus Solutions: Chapter 6.5

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Let

$$I = \int_0^3 \sqrt{x^2+9}dx \quad J = \int_1^3 \sqrt{x^2+9}dx \quad K = \int_1^4 \int \sqrt{x^2+9}dx$$

Express each of the following definite integrals in terms of I, J, and K.

1b. $\int_3^4 \sqrt{x^2+9}dx$

Solution:

$$\int_3^4 \sqrt{x^2+9}dx = K - I$$

□

1d. $\int_3^4 3\sqrt{x^2+9}dx$

Solution:

$$\int_3^4 3\sqrt{x^2+9}dx = 3(K - I)$$

□

1f. $\int_3^1 \sqrt{x^2+9}dx$

Solution:

$$\int_3^1 \sqrt{x^2+9}dx = -J$$

□

1h. $\int_4^3 8\sqrt{x^2+9}dx$

Solution:

$$\int_4^3 8\sqrt{x^2+9}dx = -8(K - I) = 8(I - K)$$

□

1j. $\int_0^3 7\sqrt{x^2+9}dx$

Solution:

$$\int_0^3 7\sqrt{x^2+9}dx = 7I$$

□

1l. $\int_0^4 (-5)\sqrt{x^2+9}$

Solution:

$$\int_0^4 (-5)\sqrt{x^2+9} = -5(I-J+K) = 5(J-I-K)$$

□

Let

$$P = \int_a^b p(x)dx \quad Q = \int_a^b q(x)dx \quad R = \int_a^b r(x)dx$$

Express each of the following definite integrals in terms of P, Q and R.

Solution:
2b. $\int_a^b 6q(x)dx$

Solution:

$$\int_a^b 6q(x)dx = 6Q$$

□

2d. $\int_a^b [q(x)-r(x)]dx$

Solution:

$$\int_a^b [q(x)-r(x)]dx = Q-R$$

□

2f. $\int_a^b [r(x)-4p(x)-4q(x)]dx$

Solution:

$$\int_a^b [r(x)-4p(x)-4q(x)]dx = R-4P-4Q$$

□

2h. $\int_a^b [5r(x) + 2q(x)]dx$

Solution:

$$\int_a^b [5r(x) + 2q(x)]dx = 5R + 2Q$$

□

2j. $\int_a^b [4q(x) - r(x)]dx$

Solution:

$$\int_a^b [4q(x) - r(x)]dx = 4Q - R$$

□

2l. $\int_a^a [4r(x) + q(x) - p(x)]dx$

Solution:

$$\int_a^a [4r(x) + q(x) - p(x)]dx = 0$$

□

For each function and interval given, find the average value of f over $[a, b]$, and find a number z satisfying the Mean Value Theorem for Integrals. If no such z exists, explain why.

Solution:
3b. $f(x) = x + 1$, $[1, 5]$

Solution:
We find the average value to be

$$\frac{1}{4}\int_1^5 (x + 1)dx = 4$$

The point $z = 3$ satisfies the mean value theorem for integrals.

□

3d. $f(x) = \sqrt{1 - x^2}$, $[-1, 1]$

Solution:
We find the average value to be

$$\frac{1}{2}\int_{-1}^1 \sqrt{1 - x^2}dx = \frac{\pi}{/}4$$

The points $z = \pm\frac{\sqrt{16-\pi^2}}{4}$ satisfies the mean value theorem for integrals.

□

3f. $f(x) = \lfloor x \rfloor$, $[0, 2.5]$

Solution:
We find the average value to be

$$\frac{1}{2.5}\int_0^{2.5} \lfloor x \rfloor dx = 2/2.5 = 4/5$$

Since f is not continuous, the mean value theorem for integrals does not apply.

□

3h. $f(x) = |x|$, $[0, 3]$

Solution:
We find the average value to be

$$\frac{1}{3}\int_0^{3} |x| dx = \frac{3}{2}$$

The point $z = 3/2$ satisfies the mean value theorem for integrals.

□

4b. Find bounds for the following definite integral: $\int_{-1}^{2} x^5 dx$

Solution:
Applying theorem 94 we find

$$\left| \int_{-1}^{2} x^5 dx \right| \leq \int_{-1}^{2} |x|^5 dx \leq \int_{-1}^{2} 32 dx = 32 \cdot 3 = 96$$

□

Use linearity properties and the properties of even and odd function to simplify the following definite integrals.

5b. $\int_{-2}^{2}(x^5 + 7x^2 - 2x + 1)dx$

Solution:

$$\int_{-2}^{2}(x^5 + 7x^2 - 2x + 1)dx = \int_{-2}^{2}(7x^2 + 1)dx = 2\int_0^{2}(7x^2 + 1)dx$$

□

5d. $\int_{-8}^{8}(x^2 + x\cos x)dx$

Solution:

$$\int_{-8}^{8}(x^2 + x\cos x)dx = 2\int_0^{8} x^2 dx$$

□

5f. $\int_{-1}^{1}(3x^4\tan x - x^2)dx$

Solution:

$$\int_{-1}^{1}(3x^4\tan x - x^2)dx = -2\int_0^1 x^2 dx$$

□

8. A rod occupying the interval $[0, 5]$ has a temperature distribution $T(x) = 100e^{-x}$. Find an integral representing the average temperature of the rod.

Solution:
Applying formula (6.49) we find the representation to be

$$\frac{1}{5}\int_0^5 100e^{-x} = 20\int_0^5 e^{-x}$$

□