Calculus Solutions: Chapter 5.3

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3. A drop of water is placed in a petri dish for culturing. After one day, it is estimated that 400 bacteria are present. After 2 days, it is estimated that 1800 bacteria are present. Assuming exponential growth, how many bacteria were in the original drop of water?

Solution:

$$B = B_0 e^{kt} \Rightarrow 400 = B_0 e^k \Rightarrow B_0 = 400e^{-k} \Rightarrow 1800 = 400e^{-k}e^{2k} \Rightarrow 4.5 = e^k$$

So $k = \ln 4.5$ and $B_0 = \frac{400}{4.5}$.

□

6. A certain kind of moth ball evaporates at a rate proportional to the volume of the moth ball, and loses half its volume in 6 weeks. If the moth ball must have at least 10% of its initial volume to be effective, how long is this kind of moth ball effective?

Solution:

$$dV/dt = -kV \Rightarrow V = Ce^{-kt} \Rightarrow \frac{1}{2} = e^{-6k} \Rightarrow k = \frac{\ln 2}{6}$$

So we solve

$$\frac{1}{10} = e^{-\frac{\ln 2}{6}t} \Rightarrow t = 19.9$$

□

10. The value of a new stock fits Newton's heating model. If the maximum possible value is $30 and the stock is worth $12 after 3 months, how much is it worth after 1 year? Assume its initial value is 0.

Solution:

$$12 = 30 - 30e^{-3k} \Rightarrow k = -(\ln \frac{3}{5})/3 \Rightarrow T(12) = \$26.11$$

□

13. Consider the logistic model $dQ/dt = dQ(L - Q)$

b) For what value of Q is the quantity growing most rapidly?

Solution:

$$Q'' = k(L - Q) - kQ = 0 \Rightarrow Q = \frac{L}{2}$$

□

16. In Dull City, population 3000, the spread of a rumor fits the logistic model. If 15 people are present when a rumor is started at 8:00 am, and 75 people have heard the rumor by 9:00 am, at what time will 2000 people have heard the rumor?

Solution:
We solve

$$\frac{(3000)(15)}{15 + (3000 - 15)e^{-3000k}} = 75$$

to get $k = .00054$ and we determine that 2000 people have heard the rumor by 11:40 am.

□

18. a) Given the metabolic half-life of 20 minutes, find the metabolic decay rate of urokinase.

Solution:

$$\frac{1}{2} = e^{-20k}$$

So $k = \frac{\ln 2}{20}$

□

b)

$$dQ/dt = U - kQ$$

c) Solving we get

$$\ln(U - kQ) = t + C \Rightarrow Q = \frac{1}{k}U + Ce^{-kt}$$

□

d) As time goes to infinity we get $L = \frac{U}{k}$

□

19. a) What temperature was the potato when Vincent took it out of the oven the first time?

Solution:

$$T = 350 - 280e^{-kt} \Rightarrow 180 = 350 - 280e^{-60k}$$

We solve for k and then $T(30)$ to get 131.8 degrees.

□

b) What temperature was the potato when Vincent put it back in the oven?

Solution:

$$T = 70 + (131.8 - 70)e^{-kt}$$

which we solve to get 122.33 degrees.
c) How long should Vincent leave the potato in the oven this time, assuming no further interruptions occur, in order for it to be cooked?

Solution:
35.12 minutes

□