

Calculus Solutions: Chapter 5.2

Aaron Peterson, Stephen Taylor

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2. Solve the differential equation subject to the given boundary condition.

b) $dy/dx = \sqrt{x}/y^3; y(0) = 0$

Solution:

$$y^3 dy = \sqrt{x} dx \Rightarrow \frac{y^4}{4} = \frac{2}{3} x^{3/2} + C \rightarrow \frac{y^4}{4} = \frac{2}{3} x^{3/2}$$

□

d) $yy' = 3(1 + x^2); y(1) = 2$

Solution:

$$\frac{1}{2} y^2 = 3x + x^3 + C \Rightarrow 2 = 4 + C \Rightarrow \frac{1}{2} y^2 = 3x + x^3 - 2$$

□

f) $y' = x \cos^2 y; y(0) = \pi/4$

Solution:

$$\sec^2 y dy = x dx \Rightarrow \tan y = \frac{1}{2} x^2 + C \Rightarrow 1 = C \Rightarrow \tan y = \frac{1}{2} x^2 + 1$$

□

h) $1 - y^2 y' = x^3; y(0) = 2$

Solution:

$$y^2 dy = (1 - x^3) dx \Rightarrow \frac{1}{3} y^3 = x - \frac{1}{4} x^4 + C \Rightarrow C = 8/3 \Rightarrow \frac{1}{3} y^3 = x - \frac{1}{4} x^4 + \frac{8}{3}$$

□

3b. Suppose that the population of a certain state was 650,000 in 1960 and 780,000 in 1970. If the population of the state increased at a rate proportional to the population itself, when did the state's population reach 1,000,000?

Solution:

$$dP/dt = kP \Rightarrow P = Ce^{kt} \Rightarrow C = 650,000 \Rightarrow P = (650,000)e^{kt} \Rightarrow \frac{780000}{650000} = e^{10k} \Rightarrow k = \ln 1.2/10$$

So we solve

$$1000000 = 650000e^{(t \ln(1.2))/10}$$

to get $t = 84.38$.

□

4. In a certain bacterial culture, the rate of growth is proportional to the $2/3$ power of the number of bacteria present. After 10 hours of incubation, there were 8000 bacteria; after 15 hours, there were 15,625 bacteria. With how many bacteria did the culture start?

Solution:

$$dB/dt = kB^{2/3} \Rightarrow 3B^{1/3} = kt + C \Rightarrow C = 60 - 10k \Rightarrow 3B^{1/3} = kt + 60 - 10k \Rightarrow 75 = 5k - 60 \Rightarrow k = 3$$

So we solve

$$3B^{1/3} = 3(0) + 30$$

to get $B = 1000$.

□

10. Suppose that the acceleration of a certain object falling through a viscous medium is inversely proportional to the square root of the velocity. If the object falls 16 millimeters in the first second, how far does it fall in 8 seconds?

Solution:

$$a(t) = dv/dt = \frac{k}{\sqrt{v}} \Rightarrow v = \left(\frac{3}{2}kt\right)^{2/3} \Rightarrow s(t) = \frac{3}{5}\left(\frac{3}{2}kt\right)t^{5/3} \Rightarrow s(t) = 16t^{5/3}$$

So $s(8) = 512$.

□

12. A certain boat has an outboard motor that gives a maximum thrust of 120 - $.5v$ pounds at speed v miles per hour. The resistive force of the water is

proportional to the boat's speed and is 15 pounds when the speed is 10 miles per hour. If the boat weighs 800 pounds, what is its maximum speed?

Solution:

$$dv/dt = 0 \Rightarrow 120 - 2v = 0 \Rightarrow v = 60$$

□

14.

$$dV/dt = \pi(576)(dh/dt) = -\pi\sqrt{768h}$$

which we solve to get 288 seconds.

□