Calculus Solutions: Chapter 4.9

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1. Find the limit:

b)

$$\lim_{x\to 3} \frac{1/x - 1/3}{x-3} = \lim_{x\to 3} \frac{-1/x^2}{1} = -\frac{1}{9}$$

□

d)

$$\lim_{x\to \pi/2} \frac{x-\pi/2}{\cos x} = \lim_{x\to \pi/2} \frac{1}{-\sin x} = -1$$

□

f)

$$\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} = \lim_{x\to\infty} \frac{1/x}{\frac{1/2}{\sqrt{x}}} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0$$

□

h)

$$\lim_{x\to\infty} \frac{\ln(\ln x)}{\ln x} = \lim_{x\to\infty} \frac{\frac{1}{x\ln x}}{\frac{1}{x}} = 0$$

□

j)

$$\lim_{x\to\infty} \frac{x^2-9}{3x^2+7x-3} = \frac{1}{3}$$

□

l)

$$\lim_{x\to 0+} \frac{1}{x\ln x} = \lim_{x\to 0+} \frac{1/x}{\ln x} = \lim_{x\to 0+} \frac{-1/x^2}{1/x} = \lim_{x\to 0+} \frac{-1}{x} = -\infty$$

□

2. Find the limit:

b)

$$\lim_{x\to\infty} e^{-x}\ln x = \lim_{x\to\infty} \frac{1/x}{e^x} = 0$$

□

d)

$$\lim_{x\to 0+} (\cos x)^{1/x} = e^{\lim_{x\to 0+} \frac{\ln\cos x}{x}} = e^{\lim x\to 0+ -\frac{\sin x}{\cos x}} = e^0 = 1$$

□

f)

$$\lim_{x\to 0-} (1+x)^(1/x) = e^{\lim_{x\to 0-} \frac{\ln(1+x)}{x}} = e^{\lim_{x\to 0-} \frac{1}{1+x}} = e$$

□

h)

$$\lim_{x\to 0+} (2x+1)^{(1/x)} = e^{\lim_{x\to 0+} \frac{\ln(2x+1}{x}} = e^{\lim_{x\to 0+} \frac{2}{2x+1}} = e^2$$

□

j)

$$\lim_{x\to 0+} (\sin x)^{1/x} = e^{\lim_{x\to 0+} \frac{\ln\sin x}{x}} = \lim_{x\to\infty} e^x = 0$$

□

l)

$$\lim_{x\to\infty} (1+x)^{1/x^2} = e^{\lim_{x\to\infty} \frac{\ln(1+x)}{x^2}} = e^{\lim_{x\to\infty} \frac{1/(1+x)}{2x}} = e^0 = 1$$

□

3. Find the limit:

b)

$$\lim_{x\to\infty}(2x-\sqrt{4x^2-5x}) = \lim_{x\to\infty}\frac{5x}{2x+\sqrt{4x^2-5x}} = \lim_{x\to\infty}\frac{5}{2+\sqrt{4-\frac{5}{x}}} = \frac{5}{4}$$

□

d)

$$\lim_{x\to\infty}(\sqrt{x^2-8x}-x) = \lim_{x\to\infty}\frac{-8x}{\sqrt{x^2-8x}+x} = \lim_{x\to\infty}\frac{-8}{\sqrt{1-\frac{8}{x}}+1} = -4$$

□

4b. Find the limit:

$$\lim_{x\to 0}\frac{\sin^2 x\ln(1+x)}{x(1-\cos x)} = \lim_{x\to 0}\frac{\sin^2 x}{1-\cos x}\lim_{x\to 0}\frac{\ln(1+x)}{x} = \lim_{x\to 0}\frac{2\sin x\cos x}{\sin x}\lim_{x\to 0}\frac{1}{1+x} = 2$$

□

6. Show that L'Hopital's rules will not help with the limits then find the limits by algebraic manipulation.

$$\lim_{x\to\infty}\frac{\sqrt{3x^2+6x}}{x+1} = \lim_{x\to\infty}\frac{6x+6}{(2\sqrt{3x^2+6x})} = \lim_{x\to 0}\frac{3}{\frac{6x+6}{(2\sqrt{3x^2+6x})}}$$

and we see that the pattern repeats itself therefore L'Hopital's rule does not help. Now

$$\lim_{x\to\infty}\frac{\sqrt{3x^2+6x}}{x+1} = \lim_{x\to\infty}\frac{\sqrt{3+\frac{6}{x}}}{\frac{1}{x}+1} = \sqrt{3}$$

$$\lim_{x\to\infty}\frac{5x+3\cos x}{2x} = \lim_{x\to\infty}\frac{5-3\sin x}{2}$$

which does not exist. But we have

$$\lim_{x\to\infty}\frac{5x+3\cos x}{2x} = \lim_{x\to\infty}\left(\frac{5}{2}+\frac{3\cos x}{2x}\right) = \frac{5}{2}$$