Calculus Solutions: Chapter 4.7

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2. How is the area of a circle changing when the radius is 10cm and decreasing at the rate of 3 millimeters per second?

Solution:

$$A = \pi r^2 \Rightarrow A'(r) = 2\pi r \frac{dr}{dt}$$

So the solution is $20(.3)\pi$ centimeters/second.

□

6. A kite is flying at an angle of elevation of 30 degrees. The kite string is being taken in at the rate of $\frac{1}{2}$ foot per second. If the angle of elevation does not change, how fast is the kite losing altitude?

Solution:
We see that

$$y = s \sin \frac{\pi}{6}$$

where y and s represent altitude and string length respectively. So

$$y' = \frac{1}{2}\frac{ds}{dt} = -\frac{1}{4} \text{ foot per second}$$

□

8. A water skier is being towed at the speed of 15 meters per second. He goes over a ramp that rises 1 meter in a distance of 4 meters. How fast is the skier rising as he leaves the ramp?

Solution:
The tangent of the angle of the ramp is $\frac{1}{4}$. So we get

$$y = \frac{1}{4}x \Rightarrow y' = \frac{1}{4}x' = 3.75 \text{ meters per second}$$

□

11. A glider is being towed by a cable attached to a power winch. The glider is flying at a constant altitude of 50 feet, and the cable is being taken in at the rate of 80 feet per second. How fast is the glider moving when there are 300 feet of cable out? At what rate is the angle of elevation to the glider changing?

Solution:
Letting z be the length of the cable and x be the horizontal distance from the winch we get the equation

$$x^2 + 50^2 = z^2 \Rightarrow x\frac{dx}{dt} = z\frac{dz}{dt}$$

We determine that when $z = 300$ the glider is moving at -81.13 feet per second. Letting Θ be the angle of elevation we get

$$\Theta = \arcsin\frac{50}{z}$$

. We determine that the angle of elevation is changing at a rate of .045 radians per second.

□

12. Two ships sail from the same port at the same time. Ship A travels east at 12 knots and ship B travels northeast at 8 knots. One hour after sailing, how is the distance between the two ships changing?

Solution:
Letting a and b be the distances A and B have traveled respectively we determine that the square of the distance between the ships is

$$d^2 = (\frac{\sqrt{2}}{2}b - a)^2 + (\frac{\sqrt{2}}{2}b)^2$$

Now we make use of the fact that $b = 8t$ and $a = 12t$ to determine that $d' = 8.499$ knots.

□

18. A certain bacterial culture creates a circular colony in a Petri dish, with a density of about 8000 per square centimeter. A colony that was 0.8 centimeters in diameter at 8 am is found to be 1.4 centimeters in diameter at 11 am. Assuming that the number of bacteria increases at a constant rate, how big will the colony be at 4 pm? Assuming that the number of bacteria grows exponentially, how big will the colony be at 4 pm?

Solution:
Under the constant rate, the number of bacteria is given by the linear function

$$N = 2764.6t + 4021.04$$

$$A = \frac{N}{8000} = \pi \frac{d^2}{4}$$

So at 4 pm the colony will be 2.04 centimeters in diameter. Under the exponential rate, the number of bacteria is given by

$$N = 4021.04(1.45)^t$$

Using the same method as above we get that the colony will be 3.56 centimeters in diameter at 4 pm.

□