Calculus Solutions: Chapter 4.6

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November 10, 2006

2. Green Acres makes and sells commercial fertilizer. The graph in Figure 4.41 shows the cost function C for producing q tons of fertilizer a day and the revenue function R from the sale of q tons of fertilizer per day. Estimate from the graph the following values:

a) the fixed costs of Green Acres' operation

Solution:
About \$50.

□

b) the number of tons that must be sold to break even

Solution:
3.5 tons

□

c) the level of production at which marginal cost is a minimum

Solution:
We seek the point where the first derivative is at a minimum. Approximately 4 tons.

□

d) the level of production at which the average cost is a minimum

Solution:
We consider the slope of secant lines from $(q, C(q))$ to $(0, C(0))$ and determine that the slope is at a minimum at about 6 tons.

□

e) the level of production at which profit is at a maximum

Solution:
Approximately 7 tons

□

f) the maximum profit

Solution:
About \$250 per day.

□

4. It costs a certain manufacturer 4000 + 150n dollars to make n television sets per week. If n sets per week are produced, they will sell for $250 - .5n$ dollars per set. How many television sets per week should be produced to maximize profit? to maximize revenue?

Solution:

$$P(n) = pn - (4000 + 150n) = 250n - .5n^2 - 4000 - 150n$$

$$P'(n) = -n + 100 = 0 \Leftrightarrow n = 100$$

To maximize profit they should produce 100 television sets.

$$R(n) = pn = 250n - .5n^2$$

$$R'(n) = 250 - n = 0 \Leftrightarrow n = 250$$

To maximize revenue they should produce 250 television sets.

□

6. The world's only manufacturer of left-handed widgets had determined that if q left-handed widgets are manufactured and sold per year, then the cost is $C(q) = 1000 + 5q$ dollars and the demand is $q = 280 - 8p$ widgets when the price is p. The Revenue is $R = pq$.

a) Rewrite the cost function and revenue function in terms of price p.

Solution:

$$C(p) = 1000 + 5(280 - 8p) \text{ and } R = 280p - 8p^2$$

□

b) Write the profit function P in terms of p.

Solution:

$$P = 280p - 8p^2 - (1000 + 5(280 - 8p)$$

□

c) Write the average cost function in terms of price p.

Solution:

$$AC = \frac{1000 + 5(280 - 8p)}{280 - 8p}$$

□

d) For what price is profit maximum? How many left-handed widgets should be produced at that price?

Solution:

$$P'(p) = 280 - 16p + 40 = 0 \Leftrightarrow p = 20$$

and

$$q = 120$$

e) For what price is revenue maximized? How many widgets should be produced at that price?

Solution:

$$R'(p) = 280 - 16p = 0 \Leftrightarrow p = 17.50$$

and

$$q = 140$$

□

8. Waldo runs a small hat shop. He observes that the cost of making n hats is

$$C(n) = 250 + 2n + .05n^2$$

He also notes that when he makes 12 hats per week, he can sell them for \$35 each, but when he makes 20 hats per week, he can sell them for only \$30 each. Assuming a linear demand function, how many hats should Waldo make per week for maximum profit?

Solution:
We must determine price as a function of n. We construct the linear function

$$p = -\frac{5}{8}n + 42.5$$

$$P(n) = -\frac{5}{8}n^2 + 42.5n - 250 - 2n - .05n^2$$

maximizing we see that $n = 30$

□

10. The owner of Rob's Rodents finds that he can sell 200 rats each day at the regular price of \$3.50 each. At his last annual Groundhog Day sale, he reduced the price to \$2.00 and was able to sell 320 rats. It costs Rob 18 cents per rat, and his fixed costs average \$49.00 per day.

a) If q is the number of rats offered for sale, write a cost function $C(q)$ and an average cost function $a(q)$ for Rob's Rodents.

Solution:

$$C(q) = .18q + 49$$

and

$$a(q) = .18 + \frac{49}{q}$$

□

b) Express the demand q as a function of price p, assuming it is linear.

Solution:

$$q = 480 - 80p$$

□

c) Write a revenue function $R(p)$ as a function of price for Rob's Rodents.

Solution:

$$R = 480p - 80p^2$$

□

d) What price will maximize Rob's revenue?

Solution:
Maximizing we obtain $p = \$3.00$.

□

e) Write a profit function for Rob's Rodents. What price should Rob charge to maximize profits?

Solution:

$$P = 494.4p - 80p^2 - 135.4$$

Maximizing we obtain $p = \$3.09$.

□

11. Uptown Piano Company sells 160 pianos per year; the demand is fairly constant. Inventory costs include storage of pianos on hand and shipping and handling costs when new pianos are ordered. It costs Uptown \$211 to store a piano for a year. When new pianos are ordered, it costs \$250 per piano for delivery plus \$75 per order for paper work and insurance. How many times per year, and in what lot size, should pianos be ordered to minimize inventory costs?

Solution:
Letting n be the number of pianos ordered per delivery and m be the number of orders per year we get a cost function of

$$\frac{n}{2}(211) + 250n + 75m = \frac{n}{2}(211) + 250n + \frac{(75)(160)}{n}$$

Minimizing we see that $m = 15$ and $n = 10.67$ So we order 11 ten times and 10 five times.

□

15b. If Δq is a change in quantity, the percent change in quantity is $\Delta q/q$. correspondingly, $\Delta p/p$ is the percent change in price. The elasticity of the demand is the limit of the ratio of percent change in quantity to percent change in price, or

$$E = \lim_{\Delta p \to 0} \frac{\Delta q/q}{\Delta p/p} = \frac{p}{q} \cdot \frac{dq}{dp}$$

b) $q = 50 - .3p, p = 15$

Solution:

$$E = \frac{p}{q} \cdot \frac{dq}{dp}\bigg|_{p=15} = .32967(-.3) = -.0989$$

□

17. In Exercise 10, what is the elasticity of demand for rate at the price $p = \$3.75$?

Solution:
Recall that $q = 480 - 80p$. So

$$E = \frac{3.75}{480 - 80(3.75)}(-80) = -1.66667$$

□