Calculus Solutions: Chapter 3.4

Aaron Peterson, Stephen Taylor

Differentiate the following:

3b. $y = (2x - x^2)^5$

Solution:
Applying the chain rule we have

$$y' = 5(2x - x^2)^4(2 - 2x)$$

□

3d. $y = \left(x - \frac{1}{x}\right)^3$

Solution:
Applying the chain rule we have

$$3\left(x - \frac{1}{x}\right)^2 (1 + x^{-2})$$

□

3f. $y = (1 - \cos x)^3$

Solution:
Applying the chain rule we have

$$y' = 3(1 - \cos x)^2 \sin x$$

□

3h. $y = \tan 5x$

Solution:
Applying the chain rule we have

$$y' = 5 \sec^2 5x$$

□

3j. $y = e^{x^2}$

Solution:
Applying the chain rule we have

$$y' = 2xe^{x^2}$$

□

3l. $y = 2^{\sin x}$

Solution:
Applying the chain rule we have

$$y' = 2^{\sin x} \ln 2 \cos x$$

□

Differentiate the following:

4b. $f(x) = \sec 3x \tan 4x$

Solution:
Applying the product and chain rules we find

$$f'(x) = 4 \sec 3x \sec^2 4x + 3 \tan 4x \sec 3x$$

□

4d. $f(x) = \cos^5 \pi x$

Solution:
Applying the chain rule twice we have

$$f'(x) = -5\pi \cos^4 \pi x \sin \pi x$$

□

4f. $f(x) = e^{-x} \cos 3x$

Solution:
Applying the product rule and the chain rule we find

$$f'(x) = 3e^{-x} \sin 3x - e^{-x} \cos 3x$$

□

4h. $f(x) = \cos(e^{\tan x})$

Solution:
Applying the chain rule we find

$$\begin{aligned} f'(x) &= -\sin(e^{\tan x})\frac{d}{dx}e^{\tan x} \\ &= -\sin(e^{\tan x})e^{\tan x}\sec^2 x \end{aligned}$$

□

4j. $f(x) = \frac{\cos^4 3x - \sin^4 3x}{1-\cos 3x}$

Solution:
Applying the quotient rule and the chain rule we find

$$\begin{aligned} f'(x) &= \frac{(1-\cos 3x)(-12\cos^3 3x \sin 3x - 12\sin^3 3x \cos 3x) + 3(\cos^4 3x - \sin^4 3x)(\sin 3x)}{(1-\cos 3x)^2} \\ &= \frac{3}{4}\csc^3\left(\frac{3x}{2}\right)\left[\cos\left(\frac{15x}{2}\right) - 3\cos\left(\frac{9x}{2}\right)\right] \end{aligned}$$

□

4l. $f(x) = \frac{50}{1+7e^{-0.02x}}$

Solution:
Applying the quotient rule and the chain rule we find

$$f'(x) = \frac{0.14e^{-0.02x}}{(1+7e^{-0.02x})}$$

□

Find dy/dx

5b. $y = u^2 - 2,\ u = 4x + 3$

Solution:
Applying the chain rule we find

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 8u = 8(4x+3)$$

□

5d. $y = u^2 + 4,\ u = \sin 3x$

Solution:
Applying the chain rule we find

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 6u\cos 3x = 6\cos 3x \sin 3x$$

□

5f. $y = u^3 + 1$, $u = v - 2$, $v = x^3 + 1$

Solution:
Applying the chain rule we find

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dv}\frac{dv}{dx} = 3u^2 3x^2 \\ &= 9(v-2)^2 x^2 = 9x^2(x^3-1)^2 \end{aligned}$$

□

7. Assuming that $d(e^x)/dx = e^x$, use the identity $a^x = e^{x \ln a}$ and the Chain Rule to prove that $d(a^x)/dx = a^x \ln a$.

Solution:
We compute

$$\frac{d}{dx}a^x = \frac{d}{dx}e^{x \ln a} = \ln a e^{x \ln a} = \ln a a^x$$

□

9. If $y = g(f(x))$, $f(2) = 7$, $f'(2) = -2$, and $g(x) = x^2$, find $(dy/dx)|_{x=2}$.

Solution:
By the chain rule we have

$$\frac{d}{dx}g(f(x)) = g'(f(x))f'(x)$$

Evaluating this expression as $x = 2$, where we note $g'(x) = 2x$ we find

$$g'(f(x))f'(x)\Big|_{x=2} = g'(f(2))f'(2) = -2g'(7) = -24$$

□

11. If $g'(x) = 1/x$ and $f(x) = (1 - x^3)^{-2}$, find $\frac{d}{dx}[g(f(x))]$.

Solution:
From the chain rule we note

$$\frac{d}{dx}[g(f(x))] = g'(f(x))f'(x)$$

We thus compute

$$f'(x) = -2(1-x^3)^{-3}(-3x^2) = \frac{6x^2}{(1-x^3)}$$

Thus

$$g'(f(x))f'(x) = (1-x^3)^2 \frac{6x^2}{(1-x^3)} = 6x^2(1-x^3)$$

□

13b. Use the table given on page 200 to find $\frac{d}{dx}[f(x)/g(x)]|_{x=2}$.

Solution:
We first note

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{gf' - fg'}{g^2}$$

From the table we find $f(2) = 0, f'(2) = -1, g(2) = 3, g'(2) = 2.5$. Plugging in these numbers to the above expression we find

$$\left.\frac{d}{dx}\frac{f(x)}{g(x)}\right|_{x=2} = -\frac{3}{9} = \frac{1}{3}$$

□

18. Show that the derivative of an odd function is an even function.

Solution:
Let $f(x)$ be an odd function. Then $f(x) = -f(-x)$ be definition.
Differentiating and applying the chain rule, we have

$$f'(x) = -\frac{d}{dx}f(-x) = f'(-x)$$

Thus $f'(x) = f'(-x)$ which shows the derivative of an odd function is an even function.

□