Calculus Solutions: Chapter 2.5

Aaron Peterson, Stephen Taylor

September 22, 2006

1. Find the following limits:

b) $\lim_{x\to\infty} \frac{x^2+1}{2x^2+4}$

Solution:
We have a rational function, where the degree of the numerator is the same as the degree of the denominator, so applying Theorem 35 we determine

$$\lim_{x\to\infty} \frac{x^2+1}{2x^2+4} = \frac{1}{2}$$

□

d) $\lim_{x\to\infty} \frac{x+1}{x^2-3}$

Solution:
We have a rational function, where the degree of the numerator is less than the degree of the denominator, so applying Theorem 35 we determine

$$\lim_{x\to\infty} \frac{x+1}{x^2-3} = 0$$

f) $\lim_{x\to\infty} \arctan x$

Solution:
Consider the graph of $\arctan x$ on page 75 of the text. We see that

$$\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$$

□

h) $\lim_{x\to\infty}(\sqrt{x^2+4x}-x)$

Solution:

$$\lim_{x\to\infty}(\sqrt{x^2+4x}-x)=\lim_{x\to\infty}(\sqrt{x^2+4x}-x)\left(\frac{\sqrt{x^2+4x}+x}{\sqrt{x^2+4x}+x}\right)$$

=

$$\lim_{x\to\infty}\frac{4x}{\sqrt{x^2+4x}+x}\left(\frac{\frac{1}{x}}{\frac{1}{x}}\right)=\lim_{x\to\infty}\frac{4}{\sqrt{1+\frac{4}{x}}+1}=2$$

□

5. Find the limits:

c) $\lim_{x\to-\infty}\frac{x}{x^2+1}$

Solution:

$$\lim_{x\to-\infty}\frac{x}{x^2+1}=\lim_{x\to-\infty}\frac{1}{x+1/x}=0$$

□

d) $\lim_{x\to-\infty}\arctan x$

Solution:
Consider the graph of $\arctan x$ on page 75 of the text. We see that

$$\lim_{x\to-\infty}\arctan x=-\frac{\pi}{2}$$

□

f) $\lim_{x\to\infty}\frac{1}{1+e^{-x}}$

Solution:

$$\lim_{x\to\infty}\frac{1}{1+e^{-x}}=1$$

since

$$\lim_{x\to\infty}e^{-x}=0$$

□

h) $\lim_{x\to\infty}x\sin\frac{1}{x}$

Solution:

$$\lim_{x\to\infty}x\sin\frac{1}{x}=\lim_{x\to0+}\frac{1}{x}\sin x=1$$

□

8. Find the horizontal asymptotes of the following functions, and sketch the graphs of the functions.

b) $f(x) = \frac{x+1}{x-1}$

Solution:

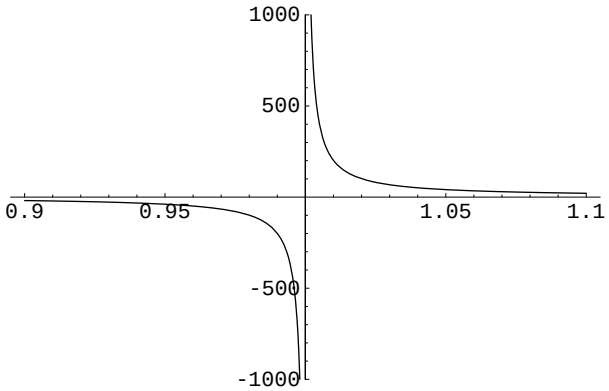

Figure 1: $\frac{1+x}{1-x}$ for $x \in [.9, 1.1]$

$$\lim_{x\to\infty} \frac{x+1}{x-1} = 1 = \lim_{x\to-\infty} \frac{x+1}{x-1}$$

So $f(x)$ has a horizontal asymptote at $y = 1$.

□

9. Find the following limits.

b) $\lim_{x\to 0^+} \frac{1}{x^3}$

Solution:
$\frac{1}{x} < \frac{1}{x^3}$ for $0 < x < 1$, so $\lim_{x\to 0^+} \frac{1}{x^3} = \infty$.

□

d) $\lim_{x\to 2^-} \frac{1}{x-2}$

Solution:
$\lim_{x\to 2^-} \frac{1}{x-2} = \lim_{x\to 0^-} \frac{1}{x} = -\infty$

□

f) $\lim_{x\to 2} \frac{1}{x-2}$

Solution:

$$\lim_{x\to 2^+} \frac{1}{x-2} = \lim_{x\to 0^+} \frac{1}{x} = \infty \neq -\infty = \lim_{x\to 2^-} \frac{1}{x-2}$$

So $\lim_{x\to 2} \frac{1}{x-2}$ does not exist.

□

10. Discuss the end behavior of the functions whose graphs are shown in a) Figure 2.34.

Solution:
Consider the graphs in Figure 2.34.
As $x \to \infty$ $f(x) \to 2$, and as $x \to -\infty$ $f(x) \to 2$.
As $x \to \infty$ $g(x) \to 0$, and as $x \to -\infty$ $g(x) \to 0$.

□

11. Determine the end behavior of each of the following functions, and sketch their graphs.

d) $f(x) = x^4 - x^3$

Solution:

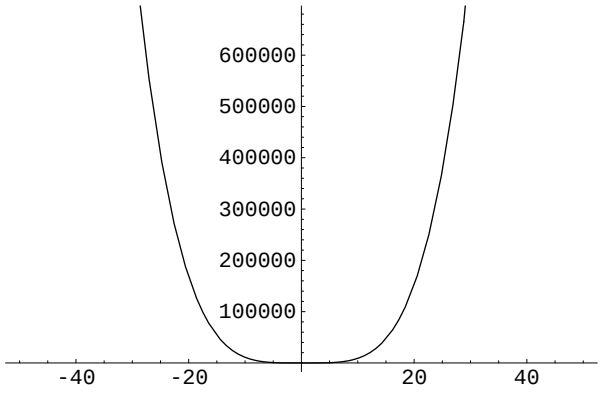

Figure 2: $x^4 - x^3$ for $x \in [-50, 50]$

Consider the above graph.

$$\lim_{x \to \infty} x^4 - x^3 = \infty$$

and

$$\lim_{x \to \infty} x^4 - x^3 = \infty$$

□

f) $f(x) = e^{-x^2}$

Solution:

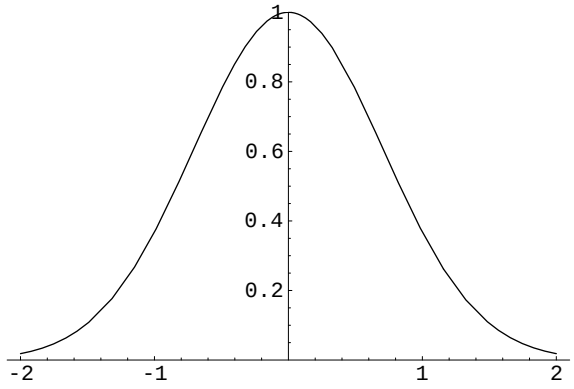

Figure 3: e^{-x^2} for $x \in [-2, 2]$

Consider the above graph.

$$\lim_{x \to \infty} e^{-x^2} = 0$$

and

$$\lim_{x \to -\infty} e^{-x^2} = 0$$

□

h) $f(x) = \frac{1}{\ln|x|}$

Solution:

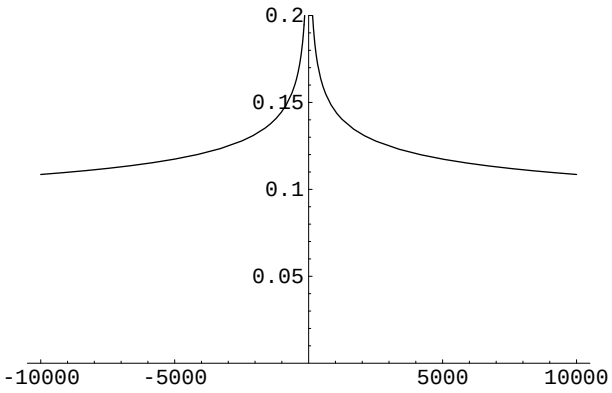

Figure 4: $\frac{1}{\ln|x|}$ for $x \in [-10000, 10000]$

Consider the above graph.

$$\lim_{x\to\infty} \frac{1}{\ln|x|} = 0$$

and

$$\lim_{x\to-\infty} \frac{1}{\ln|x|} = 0$$

□

12. Find the vertical asymptotes of each of the following functions and sketch their graphs.

Solution:
f) $f(x) = \ln(x^2 - 1)$

Solution:

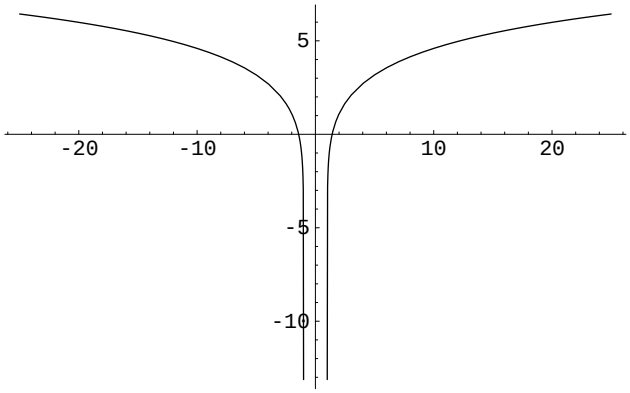

Figure 5: $\ln(x^2 - 1)$ for $x \in [-25, 25]$

Consider the above graph.

$$\lim_{x \to 1^+} \ln(x^2 - 1) = -\infty$$

and

$$\lim_{x \to -1^-} \ln(x^2 - 1) = -\infty$$

So $f(x)$ has vertical asymptotes at $x = \pm 1$.

□

13. Find the horizontal and vertical asymptotes of each of the following functions, and sketch their graphs.

f) $f(x) = \frac{e^x}{x}$

Solution:

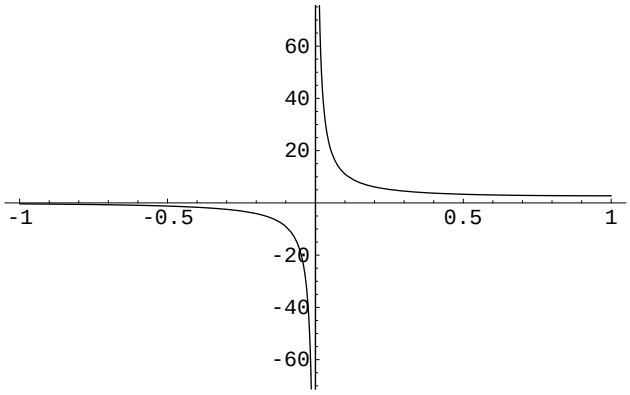

Figure 6: $\frac{e^x}{x}$ for $x \in [-1, 1]$

Consider the above graph.

$$\lim_{x \to -\infty} \frac{e^x}{x} = 0$$

So $f(x)$ has a horizontal asymptote of $y = 0$.

$$\lim_{x \to 0} \frac{e^x}{x} = \infty$$

So $f(x)$ has a vertical asymptote of $x = 0$.

□