Calculus Solutions: Chapter 2.3

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September 22, 2006

1. Find the following limits. If a limit does not exist, so state.

b) $\lim_{x\to 1}(4x-3)$

Solution:
Applying Theorem 22 we evaluate $(4x-3)$ at $x=1$. $(4)(1)-3=1$. So $\lim_{x\to 1}(4x-3)=1$.

□

d) $\lim_{x\to 3}\frac{-1}{3-x}$

Solution:
$\lim_{x\to 3}(-1)=-1$, and $\lim_{x\to 3}(3-x)=0$. So we apply Theorem 21 to determine that $\lim_{x\to 3}\frac{-1}{3-x}$ does not exist.

□

f) $\lim_{x\to -2}\frac{x-1}{x-2}$

Solution:
Since $\frac{x-1}{x-2}$ is defined at $x=-2$, we apply Theorem 23 to determine that

$$\lim_{x\to -2}\frac{x-1}{x-2}=\frac{(-2)-1}{(-2)-2}=\frac{3}{4}$$

□

h) $\lim_{x\to 2}\frac{x-1}{x-2}$

Solution:
$\lim_{x\to 2}(x-1)=1$, and $\lim_{x\to 2}(x-2)=0$. So we apply Theorem 21 to determine that $\lim_{x\to 2}\frac{x-1}{x-2}$ does not exist.

□

j) $\lim_{x\to -2} \frac{x^2-4}{x+2}$

Solution:

$$\frac{x^2-4}{x+2} = \frac{(x-2)(x+2)}{x+2} = x-2$$

So by Theorem 24,

$$\lim_{x\to -2} \frac{x^2-4}{x+2} = \lim_{x\to 2}(x-2) = -4$$

□

l) $\lim_{x\to -4} \frac{x^2-4}{x+2}$

Solution:
Since $\frac{x^2-4}{x+2}$ is defined at $x=-4$, we apply Theorem 23 to determine that

$$\lim_{x\to -4} \frac{x^2-4}{x+2} = \frac{(-4)^2-4}{(-4+2)} = -6$$

.

□

2. Find the limits if they exist. If not, so state.
b) $\lim_{x\to 0} \frac{x^2-x}{x^3+2x}$

Solution:

$$\lim_{x\to 0} \frac{x^2-x}{x^3+2x} = \lim_{x\to 0} \frac{x(x-1)}{x(x^2+2)} = \lim_{x\to 0} \frac{x-1}{x^2+2} = -\frac{1}{2}$$

□

d) $\lim_{x\to 0} \frac{x^{(3/5)}-x}{2x}$

Solution:

$$\lim_{x\to 0} \frac{x^{(3/5)}-x}{2x} = \lim_{x\to 0} \frac{x(x^{(-2/5)}-1)}{2x} = \lim_{x\to 0} \frac{x^{(-2/5)}-1}{2} = -\frac{1}{2}$$

□

f) $\lim_{x\to 1} \frac{x^2-1}{x^3-1}$

Solution:

$$\lim_{x\to 1} \frac{x^2-1}{x^3-1} = \lim_{x\to 1} \frac{(x-1)(x+1)}{(x-1)(x^2+x+1)} = \lim_{x\to 1} \frac{x+1}{x^2+x+1} = \frac{2}{3}$$

□

h) $\lim_{x\to\sqrt{2}} \frac{x-\sqrt{2}}{x^2-2}$

Solution:

$$\lim_{x\to\sqrt{2}} \frac{x-\sqrt{2}}{x^2-2} = \lim_{x\to\sqrt{2}} \frac{x-\sqrt{2}}{(x-\sqrt{2})(x+\sqrt{2})} = \lim_{x\to\sqrt{2}} \frac{1}{x+\sqrt{2}} = \frac{\sqrt{2}}{4}$$

□

j) $\lim_{x\to-1} \frac{\sqrt{3-x}-2}{x+1}$

Solution:

$$= \begin{array}{c} \lim_{x\to-1} \frac{\sqrt{3-x}-2}{x+1} = \lim_{x\to-1} \frac{(\sqrt{3-x}-2)(\sqrt{3-x}+2)}{(x+1)((\sqrt{3-x}+2)} \\ \lim_{x\to-1} \frac{-1-x}{(x+1)((\sqrt{3-x}+2)} = \lim_{x\to-1} \frac{-1}{\sqrt{3-x}+2} = -\frac{1}{4} \end{array}$$

□

m) $\lim_{x\to1} \frac{x^3-1}{x^4-1}$

Solution:

$$\lim_{x\to1} \frac{x^3-1}{x^4-1} = \lim_{x\to1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)(x^2+1)} = \lim_{x\to1} \frac{x^2+x+1}{(x+1)(x^2+1)} = \frac{3}{4}$$

□

4. Find a function f such that $\lim_{x\to c} f(x)$ does not exist for some particular number c, but $\lim_{x\to c} |f(x)|$ does exist.

Solution:
Consider the function $f(x) = -1$ for $x < 0$ and $f(x) = 1$ for $x \geq 0$.

$$\lim_{x\to0-} f(x) = -1 \neq 1 = \lim_{x\to0+} f(x)$$

So $\lim_{x\to0} f(x)$ does not exist. But $|f(x)| \equiv 1$, so $\lim_{x\to0} |f(x)| = 1$.

□