Calculus Solutions: Chapter 2.1

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1. A tomato is tossed into the air, and its height above the ground, measured in feet, at time t seconds after the toss is given by $s(t) = 4 + 48t - 16t^2$.

b) Estimate the velocity of the tomato at $t = 1$

Solution:
First, we will compute the average velocity over the intervals [1,1.5], [1, 1.1], and [1, 1.01].

$t =$	1	1.01	1.1	1.5
$s =$	36	36.16	37.44	40
$\frac{s(t)-s(1)}{t-1}$		16	14.4	8

Now we note that as the intervals become smaller, the average velocity approaches 16 feet per second. The velocity of the tomato at $t = 1$ is approximately 16 feet per second.

□

d) Estimate the velocity of the tomato at $t = 2$

Solution:
First, we will compute the average velocity over the intervals [2,3], [2, 2.5], [2,2.1]and [2, 2.05].

$t =$	2	2.05	2.1	2.5	3
$s =$	36	35.16	34.24	24	4
$\frac{s(t)-s(2)}{t-2}$		-16.8	-17.6	-24	-32

Now we note that as the intervals become smaller, the average velocity approaches -16 feet per second. The velocity of the tomato at $t = 2$ is approximately -16 feet per second.

3. A ball rolling down an inclined trough has position $s(t)$ given by $s(t) = 2t^2 + t$, with t measured in seconds from the start of the motion and s measured in inches from the starting position. Find the average velocities of the ball during the time intervals [1,3], [1,2], [1,1.5], [1,1.1], and [1,1.05]. Estimate the instantaneous velocity of the ball at $t = 1$.

Solution:
First, we compute the average velocities over the desired intervals.

$t =$	1	1.05	1.1	1.5	2	3
$s =$	3	3.255	3.52	6	10	21
$\frac{s(t)-s(1)}{t-1}$		5.1	5.2	6	7	9

Now we note that as the intervals become smaller, the average velocity approaches 5.1 inches per second. The velocity of the ball at $t = 1$ is approximately 5.1 inches per second.

□

7. On a copy of figure 2.8, sketch and label the following:
a) A line segment whose length is $f(a)$
b) A line segment whose length is $f(b)$
c) A line segment whose length is $f(b) - f(a)$
d) A line segment whose length is $b - a$
e) A line whose slope is $\frac{f(b)-f(a)}{b-a}$

Solution:
See Figure 2.6

□

9. Estimate the slope of the tangent line to the graph of the function g shown in Figure 2.9 at the point $x = a$. Is there any point at which no tangent line to the graph of g exists?

Solution:
Consider the graph of g in Figure 2.9 on page 100 in the text. At $x = a$ the slope of the tangent line to the graph is approximately -2. No tangent line to the graph exists at the broken points of the graph.

□

11. Sketch the graph of the function $f(x) = \sin x$. Estimate the slope of the tangent line to the graph at the points $x = 0, x = \frac{\pi}{2}, x = \pi, x = \frac{3\pi}{2}, x = 2\pi$. Compare the slopes you found with the values of the cosine function at the corresponding points.

Solution:

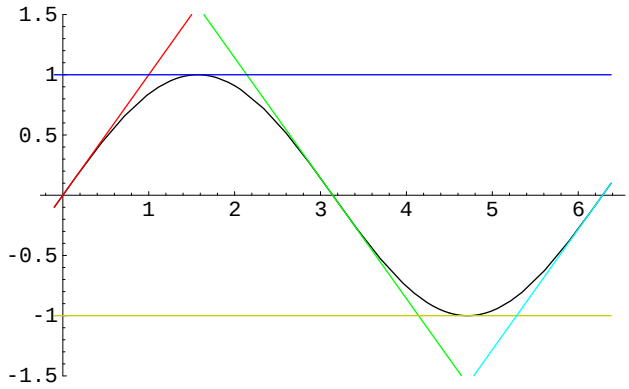

Figure 1: $\sin x$ for $x \in [0, 2\pi]$

In the above figure the red, dark blue, green, yellow, and light blue lines represent the tangent lines to the graph of $\sin x$ at the points $x = 0, x = \frac{\pi}{2}, x = \pi, x = \frac{3\pi}{2}, x = 2\pi$ respectively. The following table gives the slope of the tangent line and the value of the cosine function at a given point:

x	Tangent line at x	$\cos x$
0	1	1
$\frac{\pi}{2}$	0	0
π	-1	-1
$\frac{3\pi}{2}$	0	0
2π	1	1

Note that at each point the slope of the tangent line to $\sin x$ equals the value of $\cos x$ at that point.

□

14. The expression for the slope of the tangent line to the curve $y = f(x)$ at the point $x = a$ is

$$m_{\tan} = \lim_{b \to a} \frac{f(b) - f(a)}{b - a}.$$

Show that the substitution $b = a + h$ transforms the expression for the slope into

$$m_{\tan} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}.$$

Solution:
Note that $\lim_{b \to a} = \lim_{h \to 0}$ since $b = a + h$. Also note that $b - a = h$ for the same reason. Making the appropriate substitutions we obtain the desired result.

□