Calculus Solutions: Chapter 1.7

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Write a formula for each of the following function descriptions. Then describe its inverse and write a formula for it. It the function has no inverse function, explain why.

1b. the "multiply by two and subtract five" function

Solution:
The function is

$$f(x) = 2x - 5$$

Inverting the function, we find

$$x = \frac{f(x) + 5}{2}$$

□

1d. the "add one and cube" function

Solution:
The function is

$$f(x) = (x + 1)^3$$

Inverting the function, we find

$$x = (f(x))^{1/3} - 1$$

□

1f. the "add one and divide by twice the original" function

Solution:
The function is

$$f(x) = \frac{x + 1}{2x} = \frac{1}{2}(1 + x^{-1})$$

Inverting the function, we find

$$x = \frac{1}{2f(x) - 1}$$

□

1h. the "add two and find the fourth power" function

Solution:
The function is

$$f(x) = (x + 2)^4$$

Inverting the function, we find

$$x = (f(x))^{1/4} - 2$$

□

For each function f, determine whether f is one-to-one, and if so, find a formula for f^{-1}

2b. $f(x) = x^{1/5} - 4$

Solution:
This function is one-to-one and its inverse is given by

$$x = (f(x) + 4)^5$$

□

2d. $f(x) = 4 - x^4$

Solution:
This is an even function, and is therefore not one-to-one since $f(x) = f(-x)$ for all $x \in \mathbb{R}$.

□

2f. $f(x) = \frac{x}{3x-1}$

Solution:
This function is one-to-one and its inverse is given by

$$x = \frac{f(x)}{3f(x) - 1}$$

□

7. Show that the inverse function of the power function $f(x) = x^{\alpha}$ is $f^{-1}(x) = x^{1/\alpha}$.

Solution:
We note

$$f(f^{-1}(x)) = (x^{1/\alpha})^{\alpha} = x$$
$$f^{-1}(f(x)) = (x^{\alpha})^{1/\alpha} = x$$

which verifies the above statement.

□

10. Explain why a function is not one-to-one if a horizontal line hits its graph twice.

Solution:
Suppose a horizontal line hits a graph at two points with x-values x_1 and x_2. Then $f(x_1) = f(x_2)$ but $x_1 \neq x_2$. Since the definition of f being one-to-one is: If $x_1 \neq x_2$, $f(x_1) \neq f(x_2)$, the above violates the definition of one-to-one. Thus f is not a one-to-one function.

□