Calculus Solutions: Chapter 1.5

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Give the period of each of the following familiar periodic phenomena:

1b. the rotation of the moon around the earth.

Solution:
The period of the rotation of the moon around the earth is approximately 27.322 days.

□

1d. the rotation of the minute hand of a clock.

Solution:
The period of the rotation of the minute hand of a clock is one hour given by the clock.

□

1f. the rotation of a record going 33 1/3 rpm

Solution:
The period of the record rotation is approximately $1/33.3 \approx 0.03$ minutes or 1.8 seconds.

□

Derive each of the following identities from the previous ones.

2c. the cosine addition formula

Solution:
We note equation (1.31) states

$$\cos(s - t) = \cos s \cos t + \sin s \sin t$$

Setting $t = -t$, substitution into the previous, we find

$$\cos(s + t) = \cos s \cos(-t) + \sin s \sin(-t)$$

Noting that the cosine is an even function, and the sine is an odd function, we find the above becomes

$$\cos(s+t) = \cos s \cos t - \sin s \sin t$$

which is the desired cosine formula.

□

2e. the sine subtraction formula

Solution:
From the sine addition formula (1.35) which states

$$\sin(s+t) = \sin s \cos t + \cos s \sin t$$

we set $t = -t$ and find

$$\sin(s-t) = \sin s \cos(-t) + \cos s \sin(-t) = \sin s \cos t - \cos s \sin t$$

which is the desired result.

□

2g. the tangent subtraction formula

Solution:
We derive (1.50) by applying the previous two problems:

$$\tan(s-t) = \frac{\sin(s-t)}{\cos(s-t)} = \frac{\sin s \cos t - \cos s \sin t}{\cos s \cos t + \sin s \sin t}$$

dividing both the numerator and denominator by $\cos s \cos t$

$$\tan(s-t) = \frac{\frac{\sin s \cos t}{\cos s \cos t} - \frac{\cos s \sin t}{\cos s \cos t}}{\frac{\cos s \cos t}{\cos s \cos t} + \frac{\sin s \sin t}{\cos s \cos t}} = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

□

5. Use a right triangle that is "half" of an equilateral triangle to show how the values of $\sin \frac{\pi}{6}$, $\cos \frac{\pi}{6}$, and $\cos \frac{\pi}{3}$ are determined.

Solution:

Sketch the graphs of the following sinusoidal functions, showing at least two periods.

8b. $y = -4\sin 2x$

Solution:

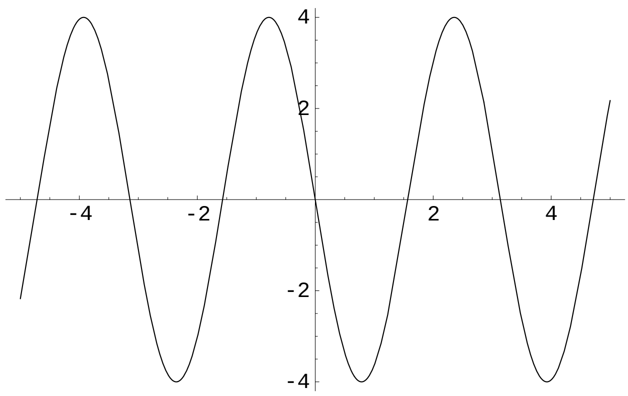

Figure 1: $y = -4\sin 2x$ for $x \in [-5, 5]$.

$\square$

8e. $y = -5\cos \pi x$

Solution:

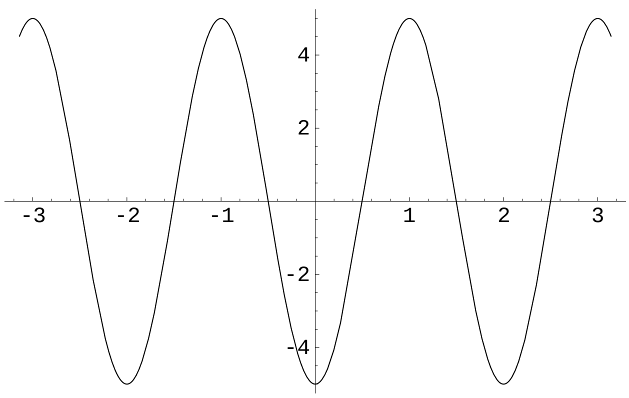

Figure 2: $y = -5\cos \pi x$ for $x \in [-\pi, \pi]$.

$\square$

8g. $y = 2\sin\frac{1}{3}x$

Solution:

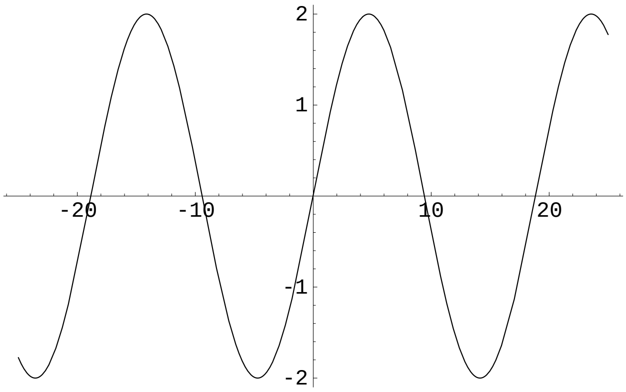

Figure 3: $y = 2\sin\frac{1}{3}x$ for $x \in [-25, 25]$.

$\square$

8i. $y = 12 + \cos\frac{1}{4}x$

Solution:

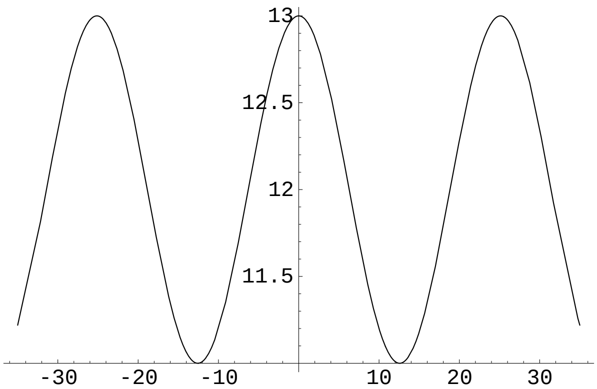

Figure 4: $y = 12 + \cos\frac{1}{4}x$ for $x \in [-35, 35]$.

$\square$

Write the formulas for the following sinusoidal curves.

9b. Figure 1.33

Solution:
The graph for this figure is

$$y = 4\sin x$$

□

9d. Figure 1.35

Solution:
This graph has a phase shift to the right of $\frac{\pi}{2}$ and a vertical shift of 1. Moreover, it has a period of π. Thus is equation is

$$y = \sin\left(2x - \frac{\pi}{2}\right) + 1$$

□

9f. Figure 1.37

Solution:
This graph has an amplitude of 24, a period of 18 and a right phase shift of $\frac{\pi}{2}$. Thus its equation is

$$y = 24\sin\left(\frac{\pi}{9}x - \frac{\pi}{2}\right)$$

□

Sketch graphs of the following functions:

13b. $y = x + \sin 2x$

Solution:

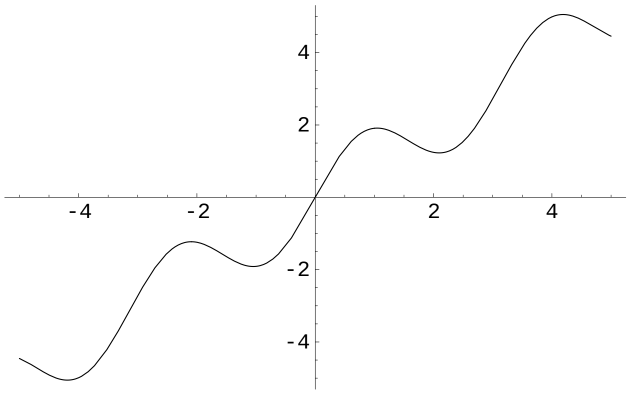

Figure 5: $y = x + \sin 2x$ for $x \in [-5, 5]$.

□

13d. $y = \sin(x^2)$

Solution:

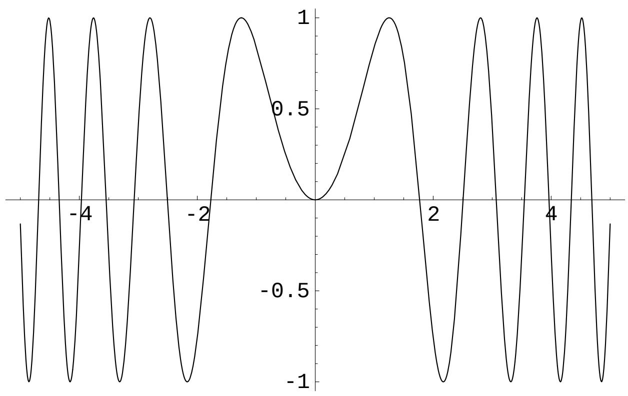

Figure 6: $y = \sin x^2$ for $x \in [-5, 5]$.

□

13h. $y = e^{\sin x}$

Solution:

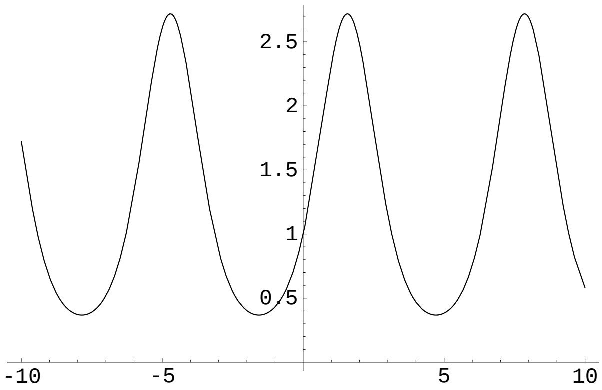

Figure 7: $y = e^{\sin x}$ for $x \in [-10, 10]$.

$\square$

21. A certain deer herd population is modeled by the function

$$P(t) = 3500 + 500 \sin \frac{\pi}{6} t + 200 \sin \frac{\pi}{3} t$$

where t is measured in months from April 1. Sketch the graph of $P(t)$ to answer the following questions.

a) When is the deer herd the largest? How large is it then?
b) When is the dear herd the smallest? How large is it then?
c) When is the heard growing most rapidly? How large is it then?

Solution:

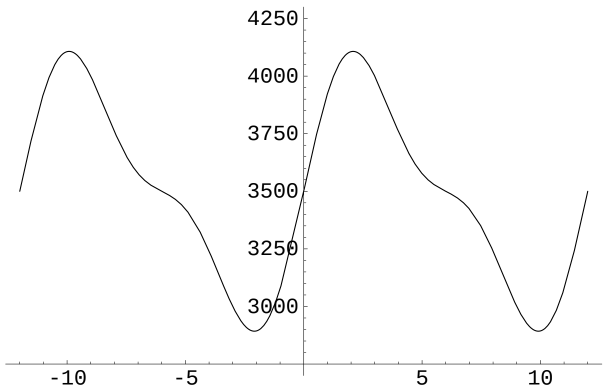

Figure 8: $y = P(t)$ for $x \in [-12, 12]$.

From the graph we see the deer herd is largest about one and a half months after April 1st, or about May 15th ($\sim$ 1400deer). We also see the dear heard is smallest about 10 months after April 1st or about February 1st (2900 deer). Finally, we note the heard is growing fastest when the "slope" of $P(t)$ is greatest; this occurs on April 1st (3500 deer). We will later use Calculus techniques to determine all of the above exactly.

□