Calculus Solutions: Chapter 1.4

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2. Show that in a regular table for an exponential function, the values have a constant ratio, while for a linear function the values have a constant difference.

Solution:
We choose $f(x) = 2^x$ for the exponential function and $g(x) = 4x + 3$ for the linear function and construct the following table:

n	$f(n) = 2^n$	$f(n+1)/f(n)$	$g(n) = 4n + 3$	$g(n+1) - g(n)$
1	2	2	7	4
2	4	2	11	4
3	8	2	15	4
4	16	2	19	4
5	32	2	23	4
6	64	2	27	4
7	128	2	31	4
8	256	2	35	4
9	512	2	39	4
10	1024	NA	43	NA

which shows the desired result.

□

4. The US population was 3.9 million in 1790 and 5.3 million in 1800.

(a) Assuming that the US population grew exponentially, find an exponential function fitting these two data points.

(b) Use this function to estimate the US population in 1860 and in 1870. Compare with the actual populations of 31.4 million and 38.6 million, respectively. Explain what you see.

Solution:
Assuming exponential growth, the world population would be modeled by the equation $y = Ab^x$. Setting year 0 to be 1790, we find

$$3.9 \cdot 10^6 = A$$

Ten years later, we find $5.3 \cdot 10^6 = 3.9 \cdot 10^6 b^{10}$. Solving, we find b has real roots $b \approx \pm 1.03115$. We take the positive value and our model becomes

$$y = 3.9 \cdot 10^6 (1.03115)^x$$

Extrapolating, we compute the following populations

$$1860 \to 3.34 \cdot 10^7$$

$$1870 \to 4.54 \cdot 10^7$$

which are slightly higher than the actual recorded values.

□

6. Tell whether the following functions could be linear or exponential, and find a formula for each.
Find formulas for the function in Figures 1.15–1.19.

7b. Figure 1.16

Solution:
This is an exponential graph. Substituting the two known points $(0, 2)$ and $(3, 6)$ into the exponential form $y = Ab^x$ we find $2 = A$, and $6 = 2b^3$. Taking the real root of the equation we find $b = 3^{1/3}$. Thus the equation for the graph is

$$y = 2 \cdot 3^{x/3}$$

□

7d. Figure 1.18

Solution:
This is an exponential decay graph. Substituting the two known points $(0, 1)$ and $(3, 0.5)$ into the exponential form $y = Ab^x$, we find $1 = A$, and $.5 = b^3$. Taking the real root we find $b = 2^{-1/3}$. Thus the equation for the graph is

$$y = 2^{-x/3}$$

□

Sketch graphs of the following exponential functions:

8b. $y = 12(1.6)^t$

Solution:

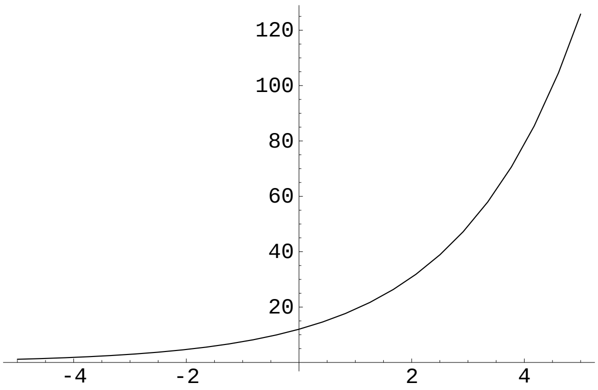

Figure 1: $y = 12(1.6)^t$ for $t \in [-5, 5]$.

$\square$

8c. $y = (2.5)^x$

Solution:

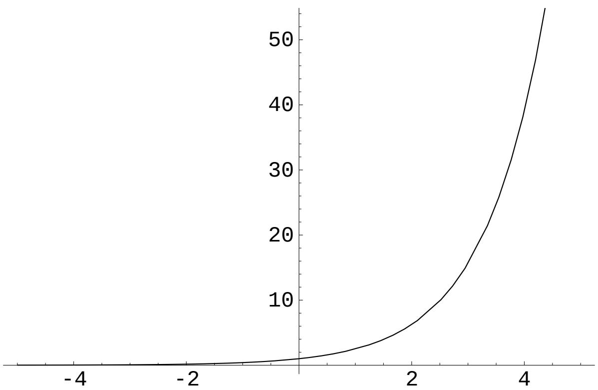

Figure 2: $y = (2.5)^x$ for $x \in [-5, 5]$.

$\square$

8e. $y = 20(0.9)^x$

Solution:

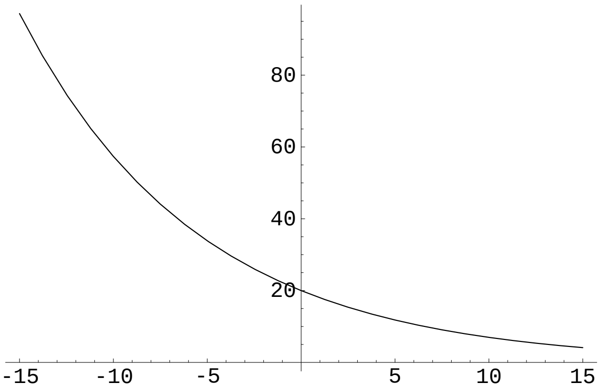

Figure 3: $y = 20(0.9)^x$ for $x \in [-15, 15]$.

$\square$

8f. $y = 2(0.5)^x$

Solution:

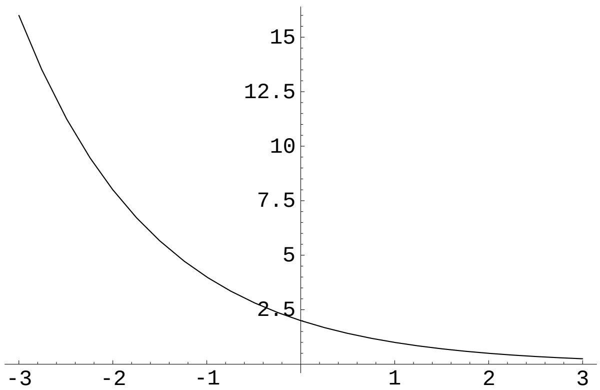

Figure 4: $y = 2(0.5)^x$ for $x \in [-3, 3]$.

$\square$

8h. $y = 5(1.5)^{-x}$

Solution:

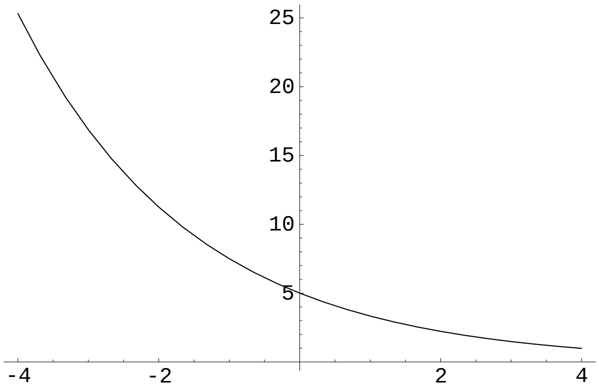

Figure 5: $y = 5(1.5)^{-x}$ for $x \in [-4, 4]$.

$\square$

Sketch graphs of the following exponential functions

9a. $y = e^{2x}$

Solution:

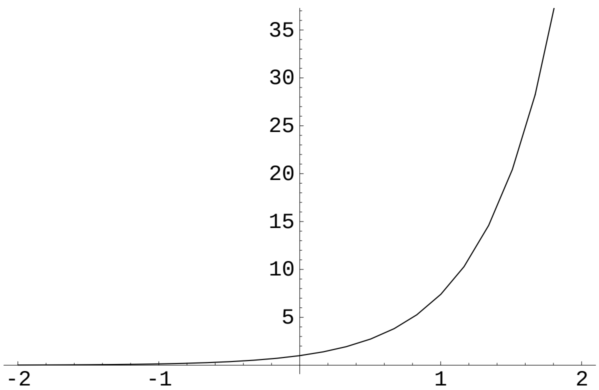

Figure 6: $y = e^{2x}$ for $x \in [-2, 2]$.

$\square$

9b. $y = e^{-2x}$

Solution:

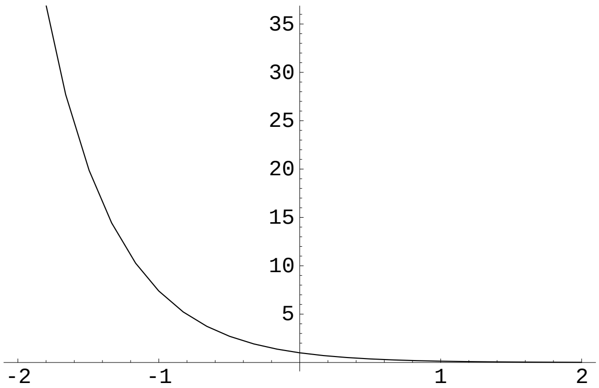

Figure 7: $y = e^{-2x}$ for $x \in [-2, 2]$.

□

9d. $y = 1 - e^{-x}$

Solution:

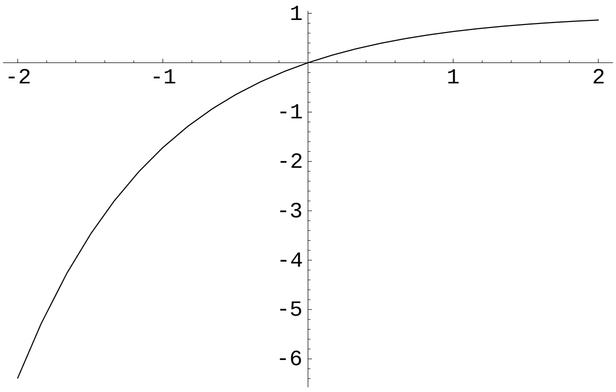

Figure 8: $y = 1 - e^{-x}$ for $x \in [-2, 2]$.

□

11. The following story appeared in 1990. Fill in the blanks so as to make the article accurate for the year in which it was published. For the third blank, assume annual compounding. (In case you are interested in what happened, the court ruled in 1991 that the family was more than 80 years too late in bringing suit, because of the statute of limitation.)

San Antonio, May 26–More than 200 years ago, a wealthy Pennsylvania merchant named Jacob DeHaven lent $450,000 to the Continental Congress to rescue the troops at Valley Forge. That loan was apparently never repaid. So Mr. DeHaven's descendants are taking the United States Government to court to collect what they believe they are owed. The total: ——- in today's dollars if the interest is compounded daily at 6 percent, the going rate at the time. If compounded yearly, the bill is only, ——-.

The descendants say that they are willing to be flexible about the amount of a settlement and that they might even accept a heartfelt thank-you or perhaps a DeHaven statue. But they also note that interest is accumulating at ——- a second.

Solution:
If the interest was compounded daily at a rate of 6%, for 200 years, the total would be

$$4.5 \cdot 10^5 \left(1 + \frac{0.06}{365}\right)^{365 \cdot 200} \approx 7.31674632846028 \cdot 10^{10}$$

If the interest was compounded yearly at a rate of 6%, for 200 years, the total would be

$$4.5 \cdot 10^5 \left(1 + \frac{0.06}{1}\right)^{1 \cdot 200} \approx 5.18067 \cdot 10^{10}$$

The increase in the amount owed in one second of time assuming interest was compounded daily would be

$$7.31674632846028 \cdot 10^{10} \left(1 + \frac{0.06}{365}\right)^{\frac{365}{365 \cdot 24 \cdot 60 \cdot 60}} \approx 7.316750013919612 \cdot 10^{10}$$

Subtracting, we find the interest is increasing at a rate of about $36,854.60 each second.

□