# Math and Music

Sound as we know is is merely a **pressure wave** generated by any vibrating object.

**Example 1.** i) A water wave

ii) Graphical examples

**Pitch** is a basic property of sound. For example, a tuba has a lower pitch than a flute.

**Example 2.** Suppose you are playing a violin. If you decrease the length of a string by placing your finger on it and then stoke is with your bow, will you

increase or decrease the pitch of the sound the instrument produces?

Plucking a string causes it to vibrate back and forth along its length as can be seen in the following pictures.

If the string vibrates up and down 100 times per second, its **frequency** is said to be 100 **cycles per second**. When you pluck only half the string, the resulting wave is half as long as the first wave, and its frequency therefore is twice as much or 200 cps.

**The higher the frequency, the higher the pitch**

For any string, plucking its full length causes it to vibrate at its **fundamental frequency**. Plucking half its length generates a wave with twice the fundamental frequency, called the first harmonic. Halving the length of the string again doubles the frequency again, so the resulting tone, called the second harmonic has four times the fundamental frequency.

Doubling the frequency of a tone raises the pitch by an **octave**.

**Example 3.** The middle C on a piano has a frequency of 260 cps. The C above middle C has a frequency of 1040 cps, whereas the C below middle C has a frequency of 130 cps.

**Scales**

The standard 12 tone scale of modern-day music is given by the following progression of notes.

$$\text{C} \to \text{C\#} \to \text{D} \to \text{D\#} \to \text{E} \to \text{F} \to \text{F\#} \to \text{G} \to \text{G\#} \to \text{A} \to \text{A\#} \to \text{B} \to \text{C}$$

Since we know that an octave jump doubles the frequency of a note, we find the linear increase in frequency from any one note to its immediate successor should be

$$f \cdot f \cdot f \cdot \cdots \cdot f = f^{12} = 2 \Rightarrow f = 2^{1/12} \approx 1.05946$$

Now consider table 11.1 on page 626, and we will play the intervals given in the table.