

Lecture 11: Chapter 4

Section 4A: Interest

September 21, 2005

On July 18, 1461, King Edward IV of England borrowed the modern equivalent of \$384 from New College of Oxford. The King soon paid back \$160, but never repaid the remaining \$224. The debt was forgotten for 535 years. Upon its rediscovery in 1996, a New College administrator wrote to the Queen of England asking for repayment, with interest. Assuming an interest rate of 4% per year, he calculated that the college was owed \$290 billion.

Simple vs Compound

Let's say you deposit \$ 1000 in a bank that promises to pay you back 2% each year. After a year, the bank sends you \$20. After then end of a second year, the bank sends you an additional \$20. After the end of the third year...

This is an example of **simple interest**.

Consider the above example. After the first year you receive \$20, but this time the bank deposits the \$20 into your account so you now have a total of \$1020. At the end of the second year the bank pays you $1020 \cdot .02 = \$20.40$. And you

know have \$ 1040.40 in your account.

This is an example of **compound interest**

In the King Edward IV example, the interest was compounded at 4% annually.

Definition 1. The **principal** in financial formulas is the balance upon which interest is paid.

Definition 2. Simple interest is interest paid only on the original principal, and not on any interest added at later days.

Definition 3. Compound interest is interest paid both on the original principal and on all interest that has been added to the original principal.

Compound Interest Formula Paid Once a Year

Definition 4. Let P be the principal investment, i be the interest rate and N the number of times interest is paid. Your accumulated balance A is

$$A = P(1 + i)^N$$

Example 1. In the New College example, $P = 224$, $i = .04$, and $N = 535$. So we find

$$A = 224 \cdot (1 + .04)^{535} = 224 \cdot 1296691085 \approx 2.9 \cdot 10^{11}$$

Example 2. You put \$10,000 in a bank who will pay you compound monthly interest of .03%. How much money will you have in one year?

Solution:

- Compound interest grow **exponentially**.

Compound Interest Formula Paid n Times a Year

If the bank pays you interest n times per year instead of one lump sum, your interest is calculated via the following formula

Let A be your total balance, P be the initial principal, i be your annual interest rate, n the number of times interest is compounded each year, and Y the total number of years. Then

$$A = P \left(1 + \frac{i}{n} \right)^{nY}$$

Example 3. Suppose you invest \$4000 at an annual interest rate of .04. If your interest is compounded monthly and you leave your money in the bank for 4 years, what will your return be at the end of the five years.

Solution:

Definition 5. The **annual percentage yield (APR)** is the actual percentage by which a balance increases in one year.

The more often interest is compounded, the higher the yield.

Continuous Compounding of Interest

What would happen if I compounded interest every second? every millisecond?
every trillionth of a second? continuously?

It turns out (and we may use a bit of basic calculus to show) that:

$$A = P \cdot e^{i \cdot Y}$$

where A is your total return, P is your principal, i is your annual interest rate, Y is the number of years you leave your money in the bank, and $e \approx 2.71828$ is Euler's constant.

Example 4. Suppose you deposit \$1000 in an account with an interest rate of %7 for 10 years where interest is compounded continuously. How much money do you receive after ten years?

Solution: