Lecture 7: Chapter 3

Section 3B: Real World Numbers

September 14, 2005

Scientific Notation

When numbers are relatively large or small, we often do not want to represent them in the traditional manner. For instance one trillion has a traditional representation: 1,000,000,000,000. We simply our representation with **scientific notation**.

Definition 1. Scientific Notation is a format in which a number is expressed as a number between 1 and 10 multiplied by a power of 10.

Example 1. (i) A B-2 stealth bomber costs approximately \$2,000,000,000 (two billion) dollars. Express this amount in scientific notation:

Solution:

2 billion dollars is expressed \$ $2 \cdot 10^9$ in scientific notation[1].

[1] Note that the exponent is equal to the number of zeros in the traditional representation

Example 2. (i) A new Ferreri you wish to buy costs 235,000 dollars. Express this number in scientific notation.

(ii) You measure the size of a cell to be 12.5 micrometers (.00000125 m). Put this value in scientific notation.

Approximations with Scientific Notation

• Scientific Notation often helps us to approximate large arithmetic calculations like[2]:

$$4958 \cdot 887 \approx 5000 \cdot 900 = (5 \cdot 10^3) \cdot (9 \cdot 10^2) = 45 \cdot 10^5 = 4500000$$

• This often comes in handy with real world problems:

Example 3. If each of the 2233169 residents of Utah use on average 76 gallons of water per day, how much water to Utahans use in one week?

Solution:

[2] the exact answer is 4397746

Putting Things in Perspective

Example 4. Adjusted for inflation, John D. Rockefeller amassed approximately 200 billion dollars. If you made $ 100 dollars each second how many years would it take to earn 200 billion dollars.

Example 5.

Item	Energy (Joules)
Energy released by metabolism of 1 candy bar	$1 \cdot 10^6$
Energy needed for 1 hour of running	$4 \cdot 10^6$
Energy released by burning 1 liter of oil	$1.2 \cdot 10^7$
Electrical energy used in a home daily	$5 \cdot 10^7$
Energy from burning 1 kilogram of coal	$1.6 \cdot 10^9$
Energy released by fission of 1 kilogram of uranium	$5.6 \cdot 10^{13}$
Energy released by fusion of hydrogen in 1 liter of water	$7 \cdot 10^{13}$
U.S. annual energy consumption	$1 \cdot 10^{20}$
World annual energy consumption	$5 \cdot 10^{20}$
Annual energy generation of Sun	$1 \cdot 10^{34}$

Example 6. If fusion of hydrogen were possible, how much water would be needed to power the United States for one year?

Solution:

Example 7. The distance from the Earth to the Sun is about 150 million kilometers. Put these numbers in perspective by using a scale model of the solar system with a 1 to 10 billion scale.

Solution:

Example 8. The distance from Earth to the nearest star outside our solar system is 4.3 light-years where a light-year can be calculated to be $9.5 \cdot 10^{12} km$. On a 1 to 10 billion scale, how far are these stars from the Earth?

Solution:

Case Studies

Example 9. How big is a billion dollars? We put this number in perspective in two ways. First consider the following:

If a professor makes \$ 50000 on average each year, how many professors could you support with one billion dollars?

Second, if you made \$ 100000 per year, how long would it take you to make one billion dollars?

Homework notes • Review the examples in this chapter. They are very good.

• In problem 21 just state which number is larger.

• On problem 57 do not scale the diameters of the planets.