Lecture 5: Chapter 2

Sections 2A, 2B: Units/Dimensions

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When we solve real world problems, our solutions will often be described by units.

Definition 1. The **units** of a quantity describe what is being measured or counted.

- We may only add or subtract numbers with the same units
- We may multiply or divide numbers with different units.

Example 1. 10 miles + 15 miles = 25 miles

10 miles/ 5 hours = 2 miles/hour

Example 2. What is the volume of a box that is ten inches wide, five inches high, and 3 inches long?

3 in $\times$ 5 in $\times$ 10 in = 150 in^3

Question: If I were asked to find the area of a circle according to the formula $A = \pi r^2$ where r is the radius measured in inches, what units would I expect the solution to have?

Question: What are the units of the volume of an object found by multiplying an area measure in meters by a depth measure in feet?

Unit Conversion

• We often need to convert between units to solve a problem.

• Making such conversions often requires that we multiply by 1 in an unorthodox manner.

For instance, in the text we find the equation:

$$1 = \frac{1}{1} = \frac{8}{8} = \frac{\frac{1}{4}}{\frac{1}{4}} = \frac{1\,\text{kilogram}}{1\,\text{kilogram}} = \frac{1\,\text{week}}{7\,\text{days}} = \frac{12\,\text{inches}}{1\,\text{foot}}$$

Note these are all equivalent representations of the number 1. If units were not present, this equation would be nonsense.

Conversion Factors

Definition 2. A **conversion factor** is a quotient of two values described by different units whose ratio is equal to the unitless number 1.

Example 3. Convert the distance of 60 inches to feet.

Solution:

$$60\text{ in} = 60\text{ in} \cdot \frac{1\text{ ft}}{12\text{ in}} = 5\text{ ft}$$

The fraction 1ft/12in is a conversion factor.

□

• We may use more than one conversion factor in a calculation:

Example 4. How many inches are in a mile?

Solution:

1 mile =

Converting Units Raised to Powers/Currency Conversions

• Suppose I want to convert square kilometers to square meters. We must alter the conversion factor as seen in the following:

Example 5. Convert 10 square kilometers into square meters.

Solution:

$$10\,\text{km}^2 = 10\,\text{km}^2 \cdot \left(\frac{1000\,\text{m}}{1\,\text{km}}\right)^2 = 10\,\text{km}^2 \cdot \frac{1000^2\,\text{m}^2}{1\,\text{km}^2} = 10 \cdot 1000^2\,\text{m} = 10^7\,\text{m}$$

□

Example 6. Convert 4400 cubic feet to cubic yards.

Solution:

$4400\,\text{ft}^3 =$

• Currency conversions are nothing but simple conversion factor problems.

Example 7. Convert \$10 into 10 British pounds.

Solution:

We look at table 2.1 on page 90 of the text to find the appropriate conversion factor. Since we are starting with units in dollars, we look at the second column in the table and perform the following calculation:

$$10\$ = 10\$ \cdot \frac{.5958\,\text{pounds}}{1\$} = 5.96\,\text{pounds}$$

Practical Problems Using Units

1. Jewelry

• A *carat* is a unit of weight defined to be exactly 0.2 gram.

• A *karat* is a measure of purity of gold based on the scale

$$x \text{ karat} = \left(\frac{x}{24} \cdot 100\right)\% \text{ pure}$$

Example 8. How much would an 18-karat gold band weighing 3.62 grams and a .65 carat diamond cost if pure gold costs 30 dollars a gram and diamond costs 2000 dollars per carat?

Solution:

How much does 18 carat gold cost per gram?

How much does the band cost?

How much does the diamond cost?

What is the total?

2. Temperature Conversion

• There are three major scales used to measure temperature: Fahrenheit, Celsius, and Kelvin.

• Note the graphic and table on page 107 that allow us to interrelate the different scales.

Example 9. Convert 90°F into temperatures on the Celsius and Kelvin scales.

Solution:

To convert Fahrenheit to Celsius, we use the formula

$$C = \frac{F - 32}{1.8} = \frac{90 - 32}{1.8} \approx 32.2^{\circ}\text{C}$$

To convert Fahrenheit to Kelvin, we use the formula

3. Light Bulb Operating Cost

• The international metric unit of **energy** is the **joule**.

• **Power** is the rate which energy is used and has units of **watts** where

$$1\,\text{watt} = 1\frac{\text{joule}}{\text{s}}$$

• A **kilowatt-hour** is defined to be:

$$1\,\text{kilowatt} - \text{hour} = 3.6 \cdot 10^6\,\text{joules}$$

Example 10. If the power company charges 8 cents per kilowatt hour how much money does it cost to keep a light on for an entire year?

Solution: How many Joules does a light use in a year?

How many kilowatt hours does this convert into?

What is the total cost?

Homework Notes: On 2A problem 13, just do parts a-d. For all currency problems, use the exchange rates given on page 90. On 2B problem 38, use the table on page 101 to obtain the conversion factors. On 2B problem 52, use the table on page 106 to obtain the conversion factor. On 2B problem 66, just do part a. On 2B problem 80, do not do the bonus.