Lecture 2: Chapter 1

Section 1B: Propositions and Truth Tables

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Now that we have examined a few fallacies, we are ready to tackle "good" arguments.

Definition 1. A **proposition** makes a claim (either an assertion or a denial) that may be either true or false. It must have the structure of a complete sentence.

- Are the following propositions?

Example 1. We are sitting in a classroom.

Example 2. Will you go to the store with me?

Truth Tables

Definition 2. Any proposition has two possible **truth values**: T = true or F = False

Definition 3. The **negation** of a proposition p is another proposition which makes the opposite claim of p. It is written *not* p and has the opposite truth value of p.

Definition 4. A **truth table** is a table with a row for each possible set of truth values for the propositions begin considered.

Example 3. Consider the following proposition p:

A truth table for p may look like this:

p	not p	not not p
T	F	
F	T	

The first column of the truth table presents the possible values of p, and the second and third column values can be found from the first. Note the third column in this example is called a double negation.

Logical Connectors (and statements)

We sometimes wish to combine propositions. Two ways of combining statements are through the words **"and"** and **"or"**.

Example 4. p = *I walked to school* q = *I attended Math 102 lecture*

Each is a separate and autonomous statement on its own. We can combine these statements to form the new statement

(p and q) = *I walked to school, and I attended Math 102 lecture*

Definition 5. Given two propositions p and q, the statement p *and* q is called their **conjunction**. It is true only if p and q are *both* true.

Truth table for the Conjunction:

p	q	p *and* q
T	T	T
T	F	F
F	T	F
F	F	F

Logical Connectors (or statements)

The word **"or"** can be used in two ways: inclusively or exclusively

Example 5. An **inclusive or:** means "either or both"

Our policy will protect against flood or fire.

Example 6. An **exclusive or:** means "one or the other, but not both."

Will you be purchasing the black car or the red one?

Definition 6. Given two propositions p and q, the statement p *or* q is called their **disjunction**. In logic, we always assume that *or* is *inclusive* unless told otherwise. Thus, the disjunction is true if either or both propositions are true, which means that it is false only when both are false

Truth table for the Disjunction:

p	q	p *or* q
T	T	T
T	F	T
F	T	T
F	F	F

(see text for examples)

If . . . Then Statements (Conditionals)

Example 7. If we travel by plane, then I will go to Alaska.

• This statement is a **conditional proposition** because it *proposes* something to be true (I will go to Alaska) on the *condition* something else is true (We travel by plane).

Definition 7. We may represent a conditional proposition in the form *if p then q*. Proposition p is called the **hypothesis** (or antecedent) and proposition q is called the **conclusion** (or consequent).

• We will construct the truth table for an if/then statement through the following example:

Example 8. If you give me $100, then I will give you $200.

We note in the example $p =$ "you give me $100" and $q =$ " I will give you $200".

Truth table for the Conditional:

p	q	*p or q*
T	T	
T	F	
F	T	
F	F	

Alternative Phrasings of the Conditional

The follows are all equivalent ways of representing the conditional statement *if p then q*:

1. *p is sufficient for q*
2. *q is necessary for p*
3. *p will lead to q*
4. *q if p*
5. *p implies q* or $p \Rightarrow q$
6. *q whenever p*

Converse, Inverse, and Contrapositive

Definition 8. Given a conditional statement *if* p*, then* q:

(i) The **converse** is given by *if* q*, then* p.

(ii) The **inverse** is given by *if not* p*, then not* q.

(iii) The **contrapositive** is given by *if not* q*, then not* p.

Example 9. Consider the conditional:

If you study, then you learn.

Converse: If you learn, then you study.

Inverse: If you do not study, then you do not learn.

Contrapositive: If you do not learn, then you do not study.

Example 10. (Class Example)

Conditional Statement:

Converse:

Contrapositive:

Truth Table for conditional, converse, inverse, and contrapositive

p	*q*	*not p*	*not q*	*if p, then q*	*if q, then p*	*if not p, then not q*	*if not q, then not p*
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Definition 9. We say that two statements are **logically equivalent** if they share the same truth values. If one is true, so is the other, and if one is false, so is the other

Important Note: The contrapositive is logically equivalent to the conditional.

This means the statement:

"If you give me \$100, then I will give you \$200"

is identical to the statement "If you do not give me \$200, then I will not give you \$100"

• Why do we care?

Homework Notes: In problems 77 and 80 just state the converse, inverse, and contrapositive of the given propositions as well as if each is true or false.