Chapter 7 Solutions

Probability

8a

10.

Solution:
This is exponential growth. After one year, the population will grow by $0.02 \cdot 10^5 = 2000$ making it 102,000; after a second year, it will grow by $0.02 \cdot 102,000 = 2040$, to 104,040; and after a third year the population will grow by a further $0.02 \cdot 104,040$; and after a third year the population will grow by a further $0.02 \cdot 104,040 = 2080.8$, which we found to 2081, making it 106,121.

□

18.

Solution:
Square 30 of the chessboard should hold $2^{29} = 536,870,912$ grains of wheat. At this point, there would be $2^{30} - 1 = 1,073,741,823$ grains of wheat on the board, weighting about 153,392 pounds.

□

20. When the chess board is full, it should weight about $2.636 \cdot 10^{15}$ pounds, or $1.3 \cdot 10^{12}$ tons, this is 659 times more than the current world harvest.

□

8b.

10.

Solution:
This makes sense.The approximate doubling time formula gives this result.

□

18.

Solution:
This is false. 96 is between 10 and 100, so $\log_{10}(90)$ is between $\log_{10} 1$ and $\log_{10}(100) = 2$.

□

22.

Solution:
This is false. 0.00045 is between 0.0001 and 0.001, and so $\log_{10}(0.00045)$ is between $\log_{10}(0.0001) = -4$ and $\log_{10}(0.001) = -3$.

□

24.

Solution:
a) $\log_{10}(50) \approx 1.699$
b) $\log_{10}(5000) \approx 3.699$
c) $\log_{10}(0.05) \approx -1.301$
d) $\log_{10}(25) \approx 1.398$
e) $\log_{10}(20) \approx -0.699$
f) $\log_{10}(5^{-2}) \approx -1.398$

□

28.

Solution:
Since prices increase by a factor of 2 in 4 weeks, and a year represents $\frac{52}{4} = 13$ doubling periods, prices will increase by a factor of $2^{13} = 8192$ in a year.

□

40.

Solution:
The doubling time formula prediction of a 1.9% per year increase increase rate is $\frac{70}{1.9} = 36.84$ years; since the rate is less than 15% the formula is valid. In a decade, oil consumption will increase by a factor of $1.019^{10} = 1.207$

□

42.

Solution:
Since 70 years is $\frac{70}{250}$ half-lives, the amount of radioactive substance decreases by a factor of $(1/2)^{0.28} = 0.824$. Since 1500 years is 6 half lives, the amount of radioactive substance decreases by a factor of $(1/2)^6 = 1/64 = 0.016$.

□

54.

Solution:
The approximate doubling time formula prediction for an 80% per month growth rate is $\frac{70}{80} = 0.875$ months. Using this, after 1 year, prices will be $2^{12}0.875 = 13,440.374$ times their current level, so what costs \$1000 today should cost $\$13,440,374$ in 1 year.

□

58.

Solution:
Since 50 years is $\frac{50}{12} = 4.16667$ half-lives, the amount of tritium decreases by a factor of $(1/2)^{4.16667}$ half-lives, the amount of tritium decreases by a factor of $(1/2)^{4.16667} = 0.05568$, so 1 kg will be reduced to about 0.055 kg. That is a significant amount of hazardous material to have to maintain for 50 years. Considering the vast scaling up which results from the considerable number o thermonuclear weapons currently in existance, we clearly need to be extremely vigilant in maintaining this hazardous material.

□

8c

10.

Solution:
This does note make sense. There are many variables in the carrying capacity, and no single model can give us a definitive answer without being based on uncertain assumptions.

□

14.

Solution:
For an annual growth rate of 1.8% the approximate doubling time formula yields $\frac{70}{1.8} = 38.89$ years. Using this, the world population will be $6 \cdot 10^9 \cdot 2^{\frac{50}{38.89}} = 14.6 \cdot 10^9$ people by 2050.

□

20.

Solution:

a) The U.S. net growth rate in 1975 was

$$\frac{14}{1000} - \frac{8.9}{1000} = \frac{5.1}{1000} = 0.0051$$

For 1985, it was

$$\frac{15.7}{1000} - \frac{8.7}{1000} = \frac{7}{1000} = 0.007$$

For 1995, it was

$$\frac{15.1}{1000} - \frac{8.8}{1000} = \frac{6.3}{1000} = 0.0063$$

b) Between 1975 and 1995, the birth rate in the U.S. increased a bit, and then decreased slightly, while the death rate was pretty constant. The population can be expected to grow at a modest rate over the next 20 years.

□

22.

Solution:
The actual growth rate when the population is 10 million is

$$0.05 \cdot \left(1 - \frac{10 \cdot 10^6}{100 \cdot 10^6}\right) = 0.045$$

When the population is 50 million, the growth rate is

$$0.05\left(1 - \frac{50 \cdot 10^6}{100 \cdot 10^6}\right) = 0.025$$

When the population is 90 million, the growth rate is

$$0.05 \cdot \left(1 - \frac{90 \cdot 10^6}{100 \cdot 10^{10}}\right) = 0.005$$

□

32.

The base growth rate for the logistic model is given by

$$\frac{0.021}{1 - \frac{3 \cdot 10^6}{20 \cdot 10^9}} = 0.0247$$

which we now use to predict the growth rate for the growth rate for the current population of 6 billion:

$$0.0247 \cdot \left(1 - \frac{6 \cdot 10^9}{20 \cdot 10^9}\right) = 0.0173$$

This is significantly more than the actual current growth rate of 1.4%. The carrying capacity is too high for this logistic model to match reality.

□

8d.

6.

Solution:
This makes sense. That is the nature of a logarithmic scale.

□

16.

Solution:
A siren at 30 meters, which has a loudness of 100dB, is 10^{10} times louder than the softest audible sound.

□

22.

Solution:
One meter away is 100 times closer than 100 meters away from the speaker. So the sound is $100^2 = 10^4$ times more intense at 1 meter away than at 100 meters away.

□

28.

Solution:
A pH of 3.5 results in a hydrogen ion concentration of

$$[H^+] = 10^{-3.5}$$

or about $3.2 \cdot 10^{-4}$ moles per liter.

□

32.

Solution:
The hydrogen ion content of the acid rain is $[H^+] = 10^{-2.5}$, while the hydrogen ion content or ordinary rain is $[H^+] = 10^{-6}$. So the acid rain is $\frac{10^{-2.5}}{10^{-6}} = 10^{3.5} \approx 3160$ times more acidic.

□

34.

Solution:
This is unbelievably loud, considerably louder than a nearby jet, and we would expect very serious effects.

□

36.

Solution:
This is a major earthquake, and we would expect very serious effects in a densely populated city such as Tokyo.

□