Chapter 7 Solutions

Probability

7a:

10.

Solution:
This does not make sense. We have no theoretical or empirical basis for making a precise statement of the probability of life on Mars.

□

14.

Solution:
There are 8 types of skis, 6 types of bindings, and 7 types of boots, so the total number of ski/binding/boot packages available is $8 \cdot 6 \cdot 7 = 336$.

□

26.

Solution:
There are 45 possible outcomes. Assuming these are equally likely outcomes, 15 of these are of interest, so the probability of selecting a blue M& M from the bag is $\frac{15}{45} \approx 0.33$.

□

30.

Solution:
Based on the assumption of 1 flood every 100 years, the empirical probability of a 100-year flood this year is $\frac{1}{100} = 0.01$.

□

42.

Solution:
If we assume that "no rain" implies a sunny day, then the probability of a sunny day is $1 - 0.2 = 0.8$.

□

44.

Solution:
There are 36 equally likely outcomes, $\{(1,1),(1,2),\ldots,(1,6),(2,1),(2,2)\ldots,(6,6)\}$. Rolling a double-6 corresponds to just one of these outcomes, namely $(6,6)$, and so the probability of rolling a double-6 is $\frac{1}{36}$, and the probability of note rolling a double-6 is $1-\frac{1}{36}\approx 0.9722$.

□

56.

Solution:
Since 4.5% of women married for the first time between the ages of 35 and 44, the probability that a randomly encountered married woman was married for the first time between the ages of 35 and 44 is 0.045. Continuing in this way, we obtain the following probability distribution for the ages for women and men

	Women	Men
Under 20	0.166	0.066
20-24	0.408	0.360
25-29	0.272	0.343
30-34	0.101	0.148
35-44	0.045	0.071
45-64	0.007	0.011
Over 65	0.001	0.001

□

7b.

8.

Solution:
Does not make sense. The first probability is the same for every possible number in the lottery, so all 6-number tickets have the same low chance of being drawn regardless of whether the numbers are in order.

□

16.

Solution:
These events are independent. The probability that both chips are defective is the product of the probabilities that each one is, i.e., $0.015\cdot 0.015=0.000225$.

$\square$

22.

Solution:
These events are non-overlapping, and the probability of getting a sum of $2, 3$ or 4 is the sum of the individual probabilities, which were found in Example 8 in Unit 7A in the text. We get $\frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \frac{1}{6} = 0.167$.

$\square$

32.

Solution:
The probability of getting rain at least once in ten days is 1 minus the probability of not getting rain on each of the ten days, namely
$1 - (1 - 0.1)^{10} = 0.651$

$\square$

38.

Solution:
Dominant and Recessive Genes. P(AA)=0.25; P(Aa)=0.5; P(aa)=0.25.
P(dominant)=P(AA or Aa)=P(AA)+P(Aa)=0.75; P(recessive)=P(aa)=0.25

$\square$

40.

Solution:
a) These events are non-overlapping, so the probability of getting a \$2,\$5 or \$10 winner is the sum of the individual probabilities, i.e.,

$$\frac{1}{10} + \frac{1}{50} + \frac{1}{500} = \frac{61}{500} = 0.122$$

which is a bit greater than the $\frac{1}{10} = 0.1$ chance of only a \$2 winner.

b) The probability of getting at least one \$5 winner in 50 tries is

$$1 - \left(1 - \frac{1}{50}\right)^{50} = 0.636$$

c) The probability of getting at least one \$10 winner in 500 tries is

$$1 - \left(1 - \frac{1}{500}\right)^{500} = 0.632$$

$\square$

7c

10.

Solution:
Does note make sense. This is the gambler's fallacy.

□

14.

Solution:
It means the person is "due" for an accident or citation. If accidents happen randomly, it is not true.

□

16.

Solution:
The probability of tossing three heads in three tosses of a fair coin is $\frac{1}{8}$, and the probability of not tossing three heads is $\frac{7}{8}$, so the expected value of the game is $(\$10 \cdot \frac{1}{8}) + (-\$1 \cdot \frac{7}{8}) = \$0.375$

□

20.

There are four events, each with a probability and value to the company: a policy purchase, a \$5000 claim, a \$10000 claim and a \$30000 claim. Thus, the expected value to the insurance company of each policy is

$$(\$500 \cdot 1) + (-\$5000 \cdot \frac{1}{50}) + (-\$10000 \cdot \frac{1}{100}) + (-\$30,000 \cdot \frac{1}{200}) = \$150$$

If the company sells 100,000 policies, its expected profit is $100,000 \cdot \$150 = \$15,000,000$.

□

24. Gambler's Fallacy and Dice

a) Even numbers have come up 45 times out of 100, so you have won \$45 but lost \$55, so you are down \$10.

b) In total, even numbers have come up $45 + 47 = 92$ times out of 200, so you have won \$92 but lost \$(200-92)=108, so you are down \$16.

c) By this stage, even numbers have come up $45 + 47 + 148 = 240$ times out of 500, so you have won \$240 but lost \$(500-240)=\$260, so you are down \$260-\$240=\$20.

d) If even numbers come up M times in the next 100 rolls, then overall they have come up $240 + M$ times in the first 600 rolls, and consequently odd numbers have come up $600 - (240 + M) = 360 - M$ times. At that stage, you would have won $\$(240 + M)$, and had to pay $\$(360 - M)$. Breaking even means that the wins and losses cancel out, so we set $240 + M = 360 - M$ and solve for M, finding that $M = 60$. This is certainly possible, although not too likely!

e) The percentages of even numbers after 100,200 and 500 rolls, respectively, were $\frac{45}{100} = 0.45 = 45\%$, $\frac{45+47}{200} = 0.46 = 46\%$ and $\frac{45+47+148}{500} = 0.48 = 48\%$, which are getting closer and closet to the predicted longterm 50% rate, despite your mounting losses.

□

32. There are ten events here, only this time the jackpot is worth \$100 million. Thus, your expected win for each ticket purchased is

$$-1 \cdot 1 + 10^8 \cdot \frac{1}{80,089,128} + 10^5 \cdot \frac{1}{1,953,393} + 5000 \cdot \frac{1}{364,042} + 100 \cdot \frac{1}{8879} + 100 \cdot \frac{1}{8466}$$

$$+7 \cdot \frac{1}{207} + 7 \cdot \frac{1}{605} + 4 \cdot \frac{1}{188} + 3 \cdot \frac{1}{74} = 0.4438$$

so you expect to win about 44 cents per ticket. Over the course of a year, your expected win is about \$161.

□

36. If we interpret the expected value of American ages to mean the expected value of the age of a randomly selected American, then we can argue that according to the given categories, there are six possible events. The first is that the person selected is under 14, with value 6.5 years, which happens with probability 0.200, since 20% of people are in that age group; the second is that the person is between 14 and 24 years of age, with value 19 years, which happens with probability 0,153; and so on. The last corresponds to a value of 75 years and happens with probability 0,126. Hence, the expected value of American ages is

$$6.5 \cdot 0.2 + 19 \cdot 0.153 + 29.5 \cdot 0.136 + 39.5 \cdot 0.163 + 54.4 \cdot 0.222 + 75 \cdot 0.126 = 36.21$$

□

38.

a) There are two possible outcomes here, a lost of 37 cents for you, or a \$1 million win. Your expected gain is:

$$(-0.37 \cdot 1) + (10^6 \cdot \frac{1}{10^7}) = -0.27$$

so you are expected to lose 27 cents.

b) In this scenario, we assume that you subscribe to the three magazines one way or another, so again there are two outcomes, a loss of 37 cents plus the $7 too much you pay for the subscriptions, or a $1 million win. Your expected gain is

$$-7.37 \cdot 1 + (10^6 - 7) \cdot \frac{1}{10^7} = -7.27$$

□

7e.

6.

Solution:
Makes sense. This is indeed a case where the combinations formula is applicable, since order does not matter for the card hands.

□

12.

Solution:

$$\frac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4 = 840$$

□

18.

Solution:

$$\frac{6!}{3!} = 120$$

□

26.

Solution:
This is arrangement without repetition (permutations) so we have $10 \cdot 9 \cdots 6 = 30,240$ different five person schedules the first night, using the ten available people.

□

28.

Solution:
This is combinations, as we are only interested in the makeup of the hand, order plays no role. So we have

$$C_4^{52} = 270,725$$

different four-card hands, using the fifty-two available cards.

□

38.

Solution:
This is combinations, as we are only interested in the makeup of the team, order plays no role. So we have

$$C_8^{20} = 125,970$$

different eight member teams, using the twenty available club members.

□

42.

Solution:
a) The first digit must be one of $\{2, 3 \ldots, 9\}$ and the remaining six can be any of the ten digits, so by the multiplication rule we have

$$8 \cdot 10 \cdot 10 \cdots 10 = 8,000,000$$

total phone numbers of seven digits. This is more than enough to serve a city of 2 million.

b) Since each exchange can serve 10^4 telephones, then assuming each telephone is serving 4 people, a single exchange can serve 40,000 people, hence two exchanges are needed to serve 80,000.

□

44.

Solution:
a) This is arrangement with repetition, and so we have 10^5 different five-digit zip codes.

b) The average number of people per zip code, assuming 270 million people and 100,000 zip codes is

$$\frac{270 \cdot 10^6}{10^5} = 2700$$

c) This is also arrangement with repetition, and so we have 10^9, or a billion, different nine-digit zip codes. Since this is almost four times as large as the U.S. population, there are more than enough for everybody to have their own personal one.

□

48.

Solution:
There are

$$C_5^{52} = 2,598,960$$

ways to select 5 cards from the 52 available, and only 4 of these hands are desirable. So the desired probability is

$$\frac{4}{2,598,960} = \frac{1}{649,740}$$

□

52.

Solution:
a) The probability that a player who gets a hit 40% of the time will get at least one hit out of four tries is 1 minus the probability that he misses all four,

$$1 - 0.6^4 = 0.8704$$

so there is an 87% change.

b) The probability that a player who gets a hit 40% of the time will get a hit in 56 consecutive games–each game consisting of four at-bats– is therefore

$$0.8704^{56} = 0.0004$$

or about 1 in 2500.

c) We need to repeat the computations in a) and b) using 0.300 instead of 0.400. The probability that a player who gets a hit 30% of the time will get at least one hit out of four tries is

$$1 - 0.7^4 = 0.7599$$

The probability that such a player will get a hit in 56 consecutive games is thus

$$0.7599^{56} = 0.0000002$$

a 1 in 5 million chance.

□