Section 3E: Homework Solutions

September 26, 2005

17. $2^3 \cdot 2^5 = 2^8 = 256$

□

18. $27^{1/3} = 3$

□

19. $3^6/3^2 = 3^4 = 81$

□

In the following solve for x.

28. $(x-2)^2 = 36$

$$(x-2)^2 = 36 \rightarrow (x-2) = \pm 6 \rightarrow x = \{-4, 8\}$$

□

30. $x^3 + 3 = 30$

$$x^3 + 3 = 30 \rightarrow x^3 = 27 \rightarrow x = 3$$

□

31. $x^{1/3} = 2$

□

$$x^{1/3} = 2 \rightarrow (x^{1/3})^3 = x = 2^3 = 8$$

For the following, calculate the balance under the given assumptions

37. Find the savings plan balance after 18 months with an APR of 6% and monthly payments of $200.

Solution:

Investing \$200 ($P = 200$) each month at 6% ($APR = 0.06$) for 18 months ($Y = 1.5$) and ($nY = 18$) gives

$$100 \cdot \frac{(1 + \frac{0.06}{12})^{18} - 1}{\frac{0.06}{12}} = \$3757.16$$

□

38. Find the savings plan balance after 24 months with an APR o 5% and monthly payments of \$250.

Solution:
Investing \$250 ($P = 250$) each month at 5% ($APR = 0.05$) for 24 months ($Y = 2$) and ($nY = 24$) gives

$$100 \cdot \frac{(1 + \frac{0.05}{12})^{24} - 1}{\frac{0.05}{12}} = \$6296.48$$

□

41. You put \$200 per month in an investment plan that pays an APR of 7%. How much money will you have after 18 years? Compare this amount to the total amount of deposits made over the time period.

Solution:
Depositing \$200 monthly for 18 years at 7% yields

$$A = \$200 \cdot \frac{(1 + \frac{0.07}{12})^{12 \cdot 18} - 1}{\frac{0.07}{12}} = \$86,144.21$$

This is about twice the total amount of deposits made over the 18 years which total \$43,200.

□

45. You want to purchase a new car in 3 years and expect the car to cost \$10,000. Your bank offers a plan with guaranteed interest rate of $APR = 5.5\%$ if you make regular monthly deposits. How much should you deposit each month to end up with \$10,000 in 3 years?

Solution:
To save up \$10,000 to buy a car, by making monthly deposits for 3 years at an APR of 5.5%, you must deposit

$$\frac{\$10,000 \cdot \frac{0.055}{12}}{(1 + \frac{0.055}{12})^{12 \cdot 3} - 1} = \$256.13$$

□

47. Suppose you are 30 years old and would like to retire at age 60. Furthermore, you would like to have a retirement fund from which you can draw an income of $50,000 per year, forever. How can you do it? Assume a constant APR of 6%.

Solution:
First, we need to figure out how much you must save by the time you reach 60. In order to draw an annual income of % 50,000 from then on, without end, if we assume an APR of 6%, then the amount saved must satisfy $A \cdot 0.06 = \$50,000$. Hence $A = \$833,333$. In order to save this much after 30 years of monthly deposits at 6%, you must deposit

$$\frac{833,333 \cdot \frac{0.06}{12}}{(1+\frac{0.06}{12})^{12 \cdot 30} - 1} = \$829.59$$

□

84. Who comes out ahead? Polly deposits $50 per month in an account with an APR of 6%, while Quint deposits $40 per month in an account with an APR of 6.5%.

Solution:
Polly deposits $50 monthly for 10 years at 6% yielding

$$A = 50 \cdot \frac{(1+\frac{0.06}{12})^{12 \cdot 10} - 1}{\frac{0.06}{12}} = \$8193.97$$

Overall she deposits $6000.

Quint deposits $40 monthly for 10 years at 6.5% yielding

$$A = 40 \cdot \frac{(1+\frac{0.065}{12})^{12 \cdot 10} - 1}{\frac{0.065}{23}} = \$6736.13$$

Overall, he deposits $4800. Thus we see that although Polly has a lower APR than Quint, she comes out ahead because her monthly payments are significantly higher than his.

□

90. Suppose you want to accumulate $50,000 for your child's college fund within the next 15 years. Would the following plan meet your goal? You deposit $ 200 per month into an account with an APR of 5%

Solution:
If you deposit $200 monthly for 15 years at 5%, you will end up with

$$A = 200 \cdot \frac{(1+\frac{0.05}{12})^{12 \cdot 15} - 1}{\frac{0.05}{12}} = \$53,457.79$$

which meets your goal.