

## Section 3E: Homework Solutions

September 23, 2005

**8.** Does the following make sense?

The polygraph test showed that the suspect was lying when he claimed to be innocent, so he must be guilty/

**Solution:**

This statement **does not make sense**. Polygraph tests are not completely accurate. An error may have occurred resulting in the potential innocence of the suspect.

□

**12.** The table below shows the passing records of two rival quarterbacks in the first half and second half of a football game. Who had the higher completion percentage in the first half? Who had the higher completion percentage in the second half? Who had the higher overall completion percentage? Explain how these results illustrate Simpson's paradox.

**Solution:**

It is clear from the table that Allen had the higher completion percentage in both each half. However, his total completion percentage was  $11/26 \approx 42.3\%$  where as Abner had a total completion percentage of  $14/31 \approx 45.2\%$ . So Abner had the higher overall average. This is a great example of Simpson's paradox, in that one person had higher percentages in fractions of a whole game, but was overall worse when total stats were compared. This paradox is a result of the fact the number of attempts were not equal for both players in either half.

□

**14.** Consider the following table comparing the grade point average and mathematics SAT scores of high school students in 1988 and 1998.

(a) In general terms, how did the SAT scores of the students in the five grade categories change between 1988 and 1998?

**Solution:**

In each instance of GPA (A+ through C), the SAT mathematics scores are lower in 1998 than in 1988.

□

(b) How did the overall average SAT score change between 1988 and 1998?

**Solution:**

The overall average increased by 10 points.

□

(c) How is this an example of Simpson's paradox?

When the whole set of students taking the mathematics portion of the SAT was partitioned into GPA classes, each class decreased in score from 1988 to 1998, but the overall average increased. This is exactly Simpson's paradox, and is a result of the fact that the number of people in each GPA classification was different.

□

17. Suppose a patient has a positive mammogram. What is the change that she really has cancer?

**Solution:**

Of the women that have a positive test result,  $95/590 \approx 16\%$  have a malignant tumor.

□

18. Suppose a patient has a negative mammogram. What is the chance that she actually does have cancer?

**Solution:**

Of the women that have negative test result,  $5/9410 \approx 0.053\%$  actually have a malignant tumor.

□

19. Suppose that a polygraph is 90% accurate. The 2000 employees of a company are given a polygraph test during which they are asked whether they use drugs. All of them deny drug use, when, in fact, 1% of the employees actually use drugs. Assume that anyone whom the polygraph operator finds untruthful is accused of lying.

(a) Verify that the entries in the table below follow from the given information and explain each entry.

**Solution:**

If the drug use rate is 1% then  $2000 \cdot .01 = 20$  people actually use drugs. and 1980 do not. If 90% test positive, then  $.9 \cdot 20 = 18$  actually use drugs. and 2 do not. Also 90% of the nonusers test negative, so  $(.9 \cdot 1980 = 1782)$  do not use drugs. Finally,  $1980 - 1782 = 98$  people who do not use drugs test positive. Overall, we find  $18 + 98 = 116$  people have lied, and  $2 + 1782 = 1784$  people have told the truth.

□

(b) How many employees are accused of lying? Of these, how many were actually lying and how many were telling the truth? What percentage of those accused of lying were falsely accused?

**Solution:**

All 216 people that tested positive are accused, but only 18 are actually lying. The other 198 people are telling the truth, so  $198/216 \approx 91.7\%$  were falsely accused.

□

(c) How many employees are found truthful? Of these, how many were actually truthful? What percentage of those found truthful really were truthful?

**Solution:**

The polygraph claims 1784 people were telling the truth where actually 1782 were. So  $1782/1784 \approx 99.89\%$  of those found telling the truth were actually telling the truth.

□

**31.** According to an analysis of the 2003 federal tax cut by the accounting firm Deloitte and Touche, a single person with a household income of \$41,000 would save \$211 in income taxes, while a single person with a household income of \$530,000 would save \$12,838 in taxes. A married couple with two children and a household income of \$41,000 would save \$1208 in income taxes, while a married couple with two children and a household income of \$530,000 would save \$13,442 in taxes.

**Solution:**

(a) Compare, in absolute and relative terms, the effects of the tax cut on a single person with a household income of \$41,000 and a single person with a household income of \$530,000.

**Solution:**

In relative terms, a \$201 tax cut for a single person making \$41,000 is about one half of one percent. A \$12,838 for a single person making \$530,000 is about 2.4%. In other words, the tax cut for the latter was nearly five times greater in relative terms.

(b) Compare, in absolute and relative terms, the effects of the tax cut on a married couple with a household income of \$41,000 and a married couple with a household income of \$530,000.

**Solution:**

In relative terms, a \$1208 tax cut for a family of four making \$41,000 is about 2.4%. A cut of \$13,442 for a family of four making \$530,000 is about 2.5%. In other words the tax cuts for families of four is nearly the same in relative terms.

□

(c) As long as you have a coherent paragraph involving some argumentation, you will receive full credit for this question.

□