Section 1D: Homework Solutions

September 9, 2005

Explain whether the following arguments are inductive or deductive.

16. If I carry a heavy load in the trunk of my car, the rear wheels squeak. If the rear wheels squeak, then I get a headache. Therefore, if I carry a heavy load in the trunk of my car, I get a headache.

Solution:
This is a deductive argument, which is constructed from a chain of conditional statements.

□

18. I have never had a bad meal at Dot's Diner, so I recommend it to everyone.

Solution:
This is an inductive argument since it is based on repeated successful experiences.

□

21. The last four times I went skiing, the traffic was light on Tuesdays and heavy on Saturdays. Weekdays must have lighter traffic than weekends.

Solution:
This is an inductive argument for the same reason as problem 18.

□

For the following, determine the truth of the premises, discuss the strength of the argument, and assess the truth of the conclusion.

24. Premise: $(-6) \times (-4) = 24$
Premise: $(-2) \times (1) = 2$
Premise: $(-27) \times (-3) = 81$
Conclusion: Whenever we multiply two negative numbers, the result is a positive number.

Solution:

All of the premises are true. Note that this argument is inductive. Inductive arguments are week in mathematics, so even though the conclusion is true in this case we determine the argument to be weak.

□

26. Premise: Bach, Buxtehude, Beethoven, Brahmn, Berlioz, and Britten are great composers.

Conclusion: All great composers have names that begin with B.

Solution:
We note in that the premises are true (there is subjectivity here but all are clearly notable composers). We determine the argument to be false by offering a **counterexample**: Mozart was a great composer whose name did not begin with B. The argument is therefore invalid, since the conclusion is false.

□

For the following, test the statements with several different numbers and state whether you think each is true or false.

51. Is it true for all numbers a and b that $a^2 + b^2 = (a + b)^2$?

Solution:
Choose $a = b = 1$. Then the above equation reads

$$1^2 + 1^2 = (1 + 1)^2$$

which gives $2 = 4$ which is clearly false. We have therefore found a counterexample to the statement and conclude that the statement is false.

□

52. Is it true for all positive integers n that

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Solution:
This is in fact a true statement, so any choice of n will return a true identity. For example, choose $n = 3$. Then we have

$$1 + 2 + 3 = \frac{3 \cdot 4}{2}$$

which yields the true identity $6 = 6$.

We now consider the deductive extra credit proof:

We consider the sum

$$1 + 2 + 3 + \cdots + (n-1) + n$$

and an identical representation in reverse order

$$n + (n-1) + \cdots + 3 + 2 + 1$$

Adding these two together we find

$$2(1+2+3+\cdots+(n-1)+n) = [1+n] + [2+(n-1)] + \cdots [(n-1)+2] + [n+1]$$

$$= [n+1] + [n+1] + \cdots [n+1]$$

Since there are n terms of $[n = 1]$ we find the above is equivalent to the equation

$$2(1 + 2 + 3 + \cdots + (n-1) + n) = n(n+1)$$

Dividing by two we obtain the desired result:

$$1 + 2 + 3 + \cdots + (n-1) + n = \frac{n(n+1)}{2}$$

□